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Abstract

We consider the effects of vertical integration on the performance of long-term and spot markets when spot prices are uncertain and agents are risk averse. We find that vertical integration impairs market performance by increasing the gap between contract and (expected) spot prices. This holds regardless of whether retail prices are fixed or linked to spot prices. Depending upon the characteristics of demand and supply, vertical integration (and long-term contracting) may increase or decrease spot-price volatility.

Keywords: vertical integration, long-term contracts, spot markets, risk aversion, electricity markets
JEL Classification codes:
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1 Introduction

In this paper we consider the effects of vertical integration on the performance of long-term and spot markets when spot prices are uncertain and agents are risk averse. Our work is motivated by concerns expressed by regulatory authorities over a tendency in deregulated electricity markets towards increased vertical integration between generation and retail. For example, the EU Commission (2006, p 135) recently wrote:

> "*Vertical integration of generation and retail within the same group reduces, all other things being equal, the need to trade on wholesale markets. In turn, this can lead to a reduction of liquidity of wholesale markets. ... Lack of liquidity can have many negative effects, such as: high volatility of prices, which increases costs for hedging ... and a lack of trust that the exchange price reflects the overall supply and demand balance in the wholesale market (reduced reliability of the price signal). A lack of liquidity may also initiate a vicious circle by creating further incentives to vertical integration because operators do not want to rely on the wholesale market for their electricity supply.*"

The academic literature on vertical integration has generally painted a more optimistic picture. Building on the seminal work of Allaz and Vila (1993), the literature typically finds that vertical integration - or long-term contracts, which in these analyses tend to be treated as one and the same - is generally pro-competitive, as it reduces the incentive to exercise market power in wholesale spot markets.[1] For example, Bushnell, Mansur and Saravia (2006), who analyse price formation in the the California, New England and PJM electricity markets, write:

> "*We find that the vertical relationships between producers and retailers play a key role in determining the competitiveness of the spot markets in the markets that we study. ... Once the known vertical arrangements are explicitly modeled as part of the Cournot equilibrium, the Cournot prices are dramatically reduced and are reasonably similar to actual prices.*"[2]

[1] There also exists a literature on the impact of vertical relations on incentives for (tacit) collusion, and here results are more ambiguous; see eg. Liski and Montero (2006) and Green and Le Coq (2006).

[2] See also Powell (1993), Green (1999), Wolak (2000), Hortaçsu and Puller (2004), Kühn and Machado (2004), Bushnell (2005), Fabra and Toro (2005), Willems (2005) and Mansur (2007). On the importance of contracts for entry see e.g. Newbery (1998).

While this literature is clearly relevant for the overall assessment of vertical integration, it does not directly address the concerns expressed by regulatory authorities, as exemplified by the above citation from the EU Commission; indeed, these analyses are conducted in deterministic setups that are ill suited to deal with issues such as market liquidity, volatility of prices and risk. We have instead chosen a setup that allows us to analyse these issues directly.[3] [4]

We take as our starting point the observation that, due to fluctuations in supply and demand conditions, participants face market risk; in particular, we concentrate on price variation in the short-term, or spot, wholesale market, which is often substantial in electricity markets.[5] We assume market participants are risk averse and so have a motive for hedging.[6] In this setting, we allow for three types of relations between generators and retailers; viz. vertical integration, long-term contracting and spot-market trade.

When retail prices are fixed, as they often are in practice, generators obtain the same sort of price hedge by vertically integrating into retailing as if they had entered long-term (fixed-price) contracts; indeed, we show that, *ceteris paribus*, when generators increase retail involvement they reduce their amount of long-term contracting by the exact same amount. Nevertheless, vertical integration does have consequences

[3]Bessembinder and Lemmon (2002) consider pricing and optimal hedging in electricity markets with forward trading; see also Allaz (1992). Aïd, Porchet and Touzi (2006) are developing this model further by adding vertical integration. Central to their analysis are assumptions of quadratic utility functions, 'strong asymmetry in terms of risk between retailers and producers' and incomplete contracting, which makes vertical integration and long-term contracts non-equivalent.

[4]There is a literature on the micro structure of financial markets, which aims at explaining price formation, (lack of) liquidity and price informativeness in such markets, see for instance Kyle (1985, 1989) and Vayanos (1999, 2001). This literature does not seem directly applicable to analysis of the issues that concern us here.

[5]A traditional explanation for vertical integration is that of assuring supplies in the presence of quantity risk, i.e. of rationing; cf. Carlton (1979). In electricity markets, however, the institutional set up is such that physical balance is ensured (except in extraordinary circumstances).

[6]Following Varian (1990), we consider the assumption of risk aversion merely as a convenient reduced-form description of firms' behaviour under uncertainty. It is commonly observed that firms owned by well-diversified stockholders take costly actions to reduce risk (including taking out insurance, dealing in forward contracts, diversifying operations across industries etc). One reason may be that they are run by managers who are themselves inherently risk averse and whose compensation schemes may result in their utility being a concave function of profits (for a textbook treatment of this issue, see for instance Ogden *et al*, 2002, pp. 86-88; for a critical appraisal of the (uncritical) application of the assumption of risk aversion, see Goldberg, 1990).

for market performance. Since independent retailers trade in both spot and contract markets, net trading in both markets is affected when retailers are integrated with generators; in particular, when retailers are under-contracted, vertical integration shifts trade (i.e. net demand) from the spot market to the contract market, and *vice versa*. The implications are that vertical integration tends to push contract and (expected) spot prices apart, reduce net spot-market trade and induce generators to adjust output.

Matters are potentially different when retail prices are linked to spot prices, as they are in some electricity markets. One may interpret this case a result of regulation (as in e.g. Denmark); alternatively, one may see this type of retail contracts as resulting from retail-market competition (as in e.g. Norway). In this case, there is no hedging motive for vertical integration; indeed, retailers would prefer to source all electricity from the spot market (unless the price there is higher than in the contract market) and, if vertical integration were to occur, generators would shift supply from the spot market to their retail business. Nevertheless, the effects of vertical integration are essentially the same; spot and contract prices are pushed apart, net trade is reduced and output levels are affected.

The result that vertical integration has similar implications for wholesale prices with fixed and variable retail prices is related to different market behaviour of retailers in the two cases. When retail prices are fixed, contract purchases provide a perfect hedge for retailers and hence they only buy in the risky spot market whenever expected prices there are lower than in the contract market. When retail prices vary with spot prices the spot market provides hedging and hence purchases in the contract market only occur when prices there are sufficiently low. It follows that in both cases the effect of vertical integration will be to reduce demand in the market where price is lower and increase demand in the market with a higher price thus driving prices in the two markets further apart.

Vertical integration not only influences the level of market prices, but also their variability or volatility. The impact on spot-price volatility turns out to depend on demand and supply conditions. Consequently, while increased vertical integration and long-term contracting affect market volatility, it is not clear that volatility will be increased by such developments.

While our analytical framework is set up so as to highlight the importance of risk and hedging motives, and therefore builds on the assumption that market participants are price takers, we also consider the impact of market power. We find that, in the spot market, risk-aversion and market-power concerns interact: compared to the case of risk-neutral and price-taking behaviour, a net seller (buyer) on the spot-market has incentives to reduce (increase) supply, both to limit exposure to risk and to exercise market power. In the contract market, incentives depend in complicated ways on how changes in contract positions affect competitor behaviour in the spot market. Therefore, the overall effect of market power on market performance cannot be determined without more detailed specification of underlying supply and demand conditions.

Our analysis is inspired by events in electricity markets, but we believe it has more general application. Specifically, the analysis is relevant for all industries in which agents face price risk from intermediate markets and where vertical integration allows market participants to circumvent these markets. Examples may include manufacturing industries that rely on volatile markets for essential raw materials, as well as retailing industries in which the supply of goods is uncertain.

2 Model

We consider a game between generators and retailers.[7] In vertically integrated companies the generation arm supplies retail directly. In addition, wholesale trade may take place through long-term contracts as well as in a spot market. Spot-market prices are volatile and since players are risk-averse they take this into account when making market decisions.

The order of events is as follows:

1. The extent of vertical integration is determined.
2. Trade takes place in the long-term contract market.
3. Bids are submitted to the spot market.
4. Uncertainty is revealed.
5. The spot market is cleared and payoffs are realised.

[7] Since in the main part of the analysis we assume market participants to be price takers there is in fact no strategic interaction. However, since we do want to allow for market power later on, we introduce the game-theoretic framework from the start; in particular, we will be using the term Subgame Perfect Equilibrium rather than the term Rational Expectations Equilibrium. Note also that as long as market participants are price takers the sequencing of wholesale markets is of no importance.

This setup captures the notion that decisions on vertical integration are taken on a longer time horizon than those on trade in wholesale markets. In turn, a portfolio of long-term contracts is in place before spot-market trade occurs. We think of the spot market as a day-ahead market, although other interpretations are possible.

There are N generating firms or generators. The cost of generator n, $n = 1, 2, ..., N$, of producing q units of output is $c_n(q)$, where c_n is a smooth, increasing and convex function, $c_n(0) = 0$, $c_n' > 0$, $c_n'' > 0$.

There are M retail "outlets". Demand at retail outlet m is given by the constant k_m, $m = 1, ..., M$. Consumers pay the retail price r. In the main part of the analysis we assume this price to be fixed. However, we subsequently consider the case in which retail price is proportional to spot price.

Let the index set of retail outlets owned by generator n be $M_n \subseteq \{1, 2, ..., M\}$. Each non-integrated retail outlet operates as an independent retailer. We denote the index set of such retail outlets by $M_0 \subseteq \{1, 2, ..., M\}$.

For generator n, denote the net amount sold in the spot market, the contract market and the retail market by y_n, $\overline{y}_n$ and $x_n = \sum_{m \in M_n} k_m$, respectively. Since these are the only ways production can be sold, the following identity must hold

$$q_n \equiv y_n + \overline{y}_n + x_n. \tag{1}$$

While output and retail sales must both be non-negative, i.e. $q_n, x_n \geq 0$, we allow for the possibility that generators buy electricity wholesale, so y_n and $\overline{y}_n$ may be negative.

Generator n obtains profit

$$\pi_n^g = r x_n + p y_n + \overline{p y}_n - c_n(q_n), \tag{2}$$

where p is the spot-market price and $\overline{p}$ is the price on long-term contracts. Generator n maximises expected utility of profits $E(U_n(\pi_n^g))$, where U_n is a concave utility function.

Non-integrated retailers have two ways of covering their electricity sales, *viz.* via the spot market and through long-term contracts. For retailer m denote the amount bought in the spot market and on long-term contracts by z_m and $\overline{z}_m$, respectively. It must then hold for each non-integrated retailer m that

$$k_m \equiv z_m + \overline{z}_m. \tag{3}$$

While retail demand must be non-negative, i.e. $k_m \geq 0$, retailers could be both buyers and sellers in wholesale markets; in particular, if retailer m is over-contracted, i.e. $\overline{z}_m > k_m$, it will be a seller on the spot market, i.e. $z_m = k_m - \overline{z}_m < 0$.

Profit of non-integrated retailer m is given by

$$\pi_m^r = rk_m - pz_m - \overline{pz}_m. \tag{4}$$

Retailer m maximises expected utility of profits $E\left(V_m\left(\pi_m^r\right)\right)$, where V_m is a concave utility function.

The equilibrium condition for the spot market is given by

$$\sum_{m \in M_0} z_m + \Phi = \sum_{n=1}^{N} y_n, \tag{5}$$

where Φ is net non-retail demand in the spot market. Net non-retail demand, which can be positive or negative, i.e. $\Phi \gtreqless 0$, may come from large consumers who operate directly in the spot market, independent power producers or trade with other markets (import/export).

Apart from representing a real element of most electricity markets, the market segment Φ serves two important modelling purposes. First, it will be the source of uncertainty in the model. Second, it introduces a price-flexible segment in the spot market; given that we restrict attention to simple strategies (*viz.* quantity-setting), and strategies have to be determined before uncertainty is resolved, such a flexible segment is necessary to allow for market clearing in all contingencies. We discuss our modelling approach in the Conclusion.

We assume that the source of stochasticity is an exogenous, aggregate shock (a random variable) θ to net non-retail demand in the spot market, Φ. Hence, the demand function Φ is of the form $\Phi\left(p, \theta\right)$, where we assume demand is decreasing in price and increasing in the aggregate demand shock:

$$\frac{\partial \Phi\left(p, \theta\right)}{\partial p} < 0 \tag{6a}$$

$$\frac{\partial \Phi\left(p, \theta\right)}{\partial \theta} > 0 \tag{6b}$$

The equilibrium condition for the contract market is

$$\sum_{m \in M_0} \overline{z}_m = \sum_{n=1}^{N} \overline{y}_n. \tag{7}$$

Below we analyse subgame perfect equilibria of the model in a series of steps. Note that no active decisions are taken by players in Stages 4 and 5; in particular, generation is fully determined when decisions on retail involvement and supply in long-term and spot markets have been made. Therefore, we start the analysis by considering spot-market behaviour at Stage 3, in the section entitled "Spot-Market Equilibrium". We proceed by considering behaviour in the market for long-term contracts at Stage 2, in the section entitled "Contract-Market Equilibrium". Subsequently, we consider vertical integration of generation and retail at Stage 1, in the section entitled "Incentives for Vertical Integration".

The analysis outlined above is conducted under the assumption that players are risk-averse price takers and retail prices are fixed; that is, retail prices do not depend on wholesale prices in either the spot or contract markets. In the section entitled "Flexible Retail Prices" we consider a case in which retail prices are linked to the spot price. We end the analysis by considering how the presence of market power might affect our results, in the section entitled "Market Power". In "Concluding remarks" we summarise results and discuss some of the underlying assumptions of our analysis.

3 Spot-Market Equilibrium

In this section we analyse Stage 3 of the game presented in the previous section, taking the extent of vertical integration and players' long-term contract positions as given.

Note that spot-market price is determined based on bids placed on the market prior to realisation of uncertainty, and the price is therefore a random variable. Since retail demand is fixed and contract positions are given at this stage retailer involvement in the spot market is residually determined by identity (3). Also, given commitments in retail, contract and spot markets, generation is determined by identity (1). Therefore, at Stage 3 generator n may be seen as choosing generation so as to maximise expected utility of profit $E\left(U_n\left(\pi_n^g\right)\right)$, taking into account that spot-market supply is residually determined by $y_n = q_n - \overline{y}_n - x_n$.

The first-order condition for generator utility maximization may be written

$$E\left(U'\left(\pi^g\right)\left[p - c'\left(q\right)\right]\right) = 0, \tag{8}$$

where we have dropped the subscript n, as we shall continue to do when there is no ambiguity. The second-order condition for maximisation of expected utility is

$$E\left(U''\left[p - c'\right]^2\right) + E\left(U'\left[-c''\right]\right) < 0, \tag{9}$$

which is clearly satisfied.

In the absence of risk aversion, i.e. when $U'(\pi) \equiv u$, where u is a positive constant, it is immediate from (8) that generators set their production so as to equalise marginal generation cost and expected spot-market price. Their position in the spot market (in particular, whether they are net buyers or sellers) then has no bearing on production decisions. Furthermore, marginal costs are equalised across generators and so production costs are minimised.

This is different, however, when generators are risk averse:

Proposition 1 [8] *At equilibrium,*

$$c'(q) \leq Ep \text{ if } y > 0, \tag{10}$$

$$c'(q) \geq Ep \text{ if } y < 0. \tag{11}$$

The inequalities are strict if U is strictly concave.

Proof. See the Appendix ■

Proposition 1 states that net sellers in the spot market underproduce; that is, they set their production such that marginal cost is lower than expected spot-market price. The reverse holds for net buyers. The intuition for this result follows from the observation that, at the margin, generators balance profits and risk. A net seller, by increasing output or spot-market supply, increases exposure to risk and so requires a positive profit margin. A net buyer, on the other hand, reduces exposure to risk when increasing output and so is willing to produce at a marginal cost that exceeds (expected) spot-market price.

Note that marginal cost may be greater or larger than expected spot-market price for individual generators even if no market power is exerted. One might therefore expect generation costs not to be minimised in the aggregate; however, as we show below, at overall equilibrium generators adjust market positions so that marginal costs equal contract price and hence become identical across producers.

We next consider how spot-market equilibrium depends on long-term contracting and vertical integration. It is difficult to get very far with such considerations unless some further structure is imposed on the problem. We do so by assuming a utility function of the constant absolute risk aversion (CARA) type, i.e. that absolute risk aversion $\rho_U = -\frac{U''}{U'}$ is identically equal to a constant. This assumption is maintained throughout the remainder of the paper unless otherwise indicated.

[8] In essence, similar results were derived by Magnusson (1969), Baron (1970) and Sandmo (1971).

Lemma 1 *At spot-market equilibrium,*

$$-1 \leq \frac{\partial y}{\partial x} = \frac{\partial y}{\partial \overline{y}} < 0, \tag{12}$$

If U is strictly concave ($\rho_U > 0$), then the first inequality is strict.

Proof. See the Appendix. ■

Thus, a generator that increases its supply in the contract or retail market will offset this increase by a reduction in spot-market supply, but less than fully.

In order to understand this result, consider first how spot trade of a risk-neutral generator depends on its contractual position. Then a higher level of contract sales would be fully offset by a reduction in spot-market sales. Note that this implies that, for a net seller in the spot market, exposure to risk is reduced. A risk averter would therefore be willing to produce at a higher level, implying a smaller reduction in spot sales. Conversely, if the generator is a buyer in the spot market, risk would be excessively increased by fully compensating for an increase in contract sales by spot purchases; consequently, a risk averter would increase production in order to reduce exposure to risk.

The next result characterises the impact of a change in net supply of generators and retailers to the spot market on the level and variability of spot-market price. We measure variability by the impact of the demand shock on price, i.e. the value of $\frac{dp}{d\theta} > 0$.

Lemma 2 *Consider an event that increases net supply of generators and retailers to the spot market, i.e. $ds = d\left[\sum_{n=1}^{N} y_n - \sum_{m \in M_0} z_m\right] > 0$. This leads to a fall in expected spot-market price while spot-price variability increases if*

$$\frac{\partial^2 \Phi}{\partial p \partial \theta}\frac{\partial \Phi}{\partial p} - \frac{\partial^2 \Phi}{\partial p^2}\frac{\partial \Phi}{\partial \theta} > 0. \tag{13}$$

Proof. See the Appendix. ■

These results have some implications for the effect of long-term contracting on spot-market equilibrium. Suppose that, for some reason (say because of vertical integration by a generator and a retailer) the balance in the long-term contract market is upset, so that net supply of contracts is reduced. In order for equilibrium in the contract market to be restored, retailers must reduce their demand and generators must increase their supply. For retailers, demand in the spot market is increased by exactly the same amount as their demand falls in the contract market (cf. (3)). However, the induced reduction in spot supply of generators is smaller in magnitude than their increase in contract positions. It follows that net supply in the spot market must decrease, but by less than the original increase in contract supply. This is formally stated in the following result.

Lemma 3 *Suppose there is a change in net supply of generators and retailers in the long-term contract market,* $d\overline{s} = \sum_{n=1} d\overline{y}_n - \sum_{m \in M_0} d\overline{z}_m$*, such that* $\frac{d\overline{y}_n}{d\overline{s}} > 0$ *all* n *and* $\frac{d\overline{z}_m}{d\overline{s}} < 0$ *all* m*. Then*

$$-1 \leq \frac{ds}{d\overline{s}} < 0. \tag{14}$$

Proof. See the Appendix. ■

These observations lead to the following proposition:

Proposition 2 *Under the assumptions of Lemma 3 a decrease in net supply in the contract market leads to a fall in expected spot-market price. Spot-price variability increases if*

$$\frac{\partial^2 \Phi}{\partial p \partial \theta} \frac{\partial \Phi}{\partial p} - \frac{\partial^2 \Phi}{\partial p^2} \frac{\partial \Phi}{\partial \theta} > 0. \tag{15}$$

■

To elucidate condition (15) we consider some parameterised examples.

Linear net non-retail demand: With a linear function of the form $\Phi(p, \theta) = A\theta - Bp$, $A, B > 0$, we get $\frac{\partial^2 \Phi}{\partial p \partial \theta} \frac{\partial \Phi}{\partial p} - \frac{\partial^2 \Phi}{\partial p^2} \frac{\partial \Phi}{\partial \theta} = 0$. Hence, increased (long-term) contract sales have no effect on price variability.

Quadratic net non-retail demand: With a function of the form $\Phi(p, \theta) = A\theta - \frac{1}{2}Bp^2$, $A, B > 0$, we get $\frac{\partial^2 \Phi}{\partial p \partial \theta} \frac{\partial \Phi}{\partial p} - \frac{\partial^2 \Phi}{\partial p^2} \frac{\partial \Phi}{\partial \theta} = AB > 0$. Hence, increased contract sales lead to increased price variability.

Constant-price-elasticity net non-retail demand: With a function of the form $\Phi(p, \theta) = A\theta p^{-\varepsilon}$, $A > 0$, we get $\frac{\partial^2 \Phi}{\partial p \partial \theta} \frac{\partial \Phi}{\partial p} - \frac{\partial^2 \Phi}{\partial p^2} \frac{\partial \Phi}{\partial \theta} = -\varepsilon A^2 \theta p^{-2(\varepsilon+1)} < 0$. Hence increased contract sales lead to decreased price variability.

Note that condition (15) is local, so the effect of a marginal increase in long-term contract sales will depend on the shape of the demand curve in the area around the equilibrium price prior to the increase.

4 Contract-Market Equilibrium

Now consider Stage 2 in the order of events, i.e. the stage where retail involvement can be regarded as fixed and known, but long-term contracts remain to be written. These contracts specify price $\overline{p}$ and quantities $\overline{z}_m$, $m \in M_0$, (for retailers) and $\overline{y}_n$, $n = 1, ..., N$ (for generators). In this section we characterise the subgame perfect equilibrium for this game taking into account equilibrium behaviour in the subsequent spot market. We also analyse how equilibrium depends on the degree of vertical integration.

We start by characterising behaviour of generators. A generator wants to determine the amount sold through long-term contracts $\overline{y}$ so as to maximise $E\left(U\left(\pi^g\left(x, \overline{y}, y^*\right)\right)\right)$, where $\pi^g\left(x, \overline{y}, y\right) = rx + \overline{py} + py - c\left(x + \overline{y} + y\right)$ and y^* is the optimal quantity bid into the spot market. We get the following result.

Proposition 3 *At equilibrium, marginal generation costs equal the price of long-term contracts:*

$$c'\left(x + \overline{y} + y^*\right) = \overline{p}. \tag{16}$$

Proof. See the Appendix. ∎

The result implies that generators must have the same marginal cost, so that total generation costs are minimised. If we were to assume identical cost functions, all generators would produce the same amount $q = x + \overline{y} + y^*$. Note that, even in this case, positions in the contract and spot market, i.e. $\overline{y}$ and y^*, would not be the same (unless the extent of vertical integration were identical).

Combining the results of Propositions 1 and 3, we have the following result.

Proposition 4 *At equilibrium, when U is strictly concave,*

$$\begin{aligned} y &> 0 \text{ if and only if } Ep > \overline{p}, && (17) \\ y &< 0 \text{ if and only if } Ep < \overline{p}. && (18) \end{aligned}$$

∎

In other words, when generators are net sellers in the spot market, expected spot-market price is above the price of long-term contracts; conversely, if generators are net buyers spot-market price is below the contract price. Note that the proof of this proposition does not rely on the CARA assumption and therefore the result holds for all strictly concave U.

We next consider retailers. Each retailer maximises expected utility $EV\left(\pi^r\right)$, where $\pi^r = rk - pz - \overline{pz}$ is retailer profit and V is a concave utility function. The following first-order condition characterises optimal retailer behaviour:

$$E\left(V'\left(\pi^r\right)\left[p - \overline{p}\right]\right) = 0. \tag{19}$$

We then find an analogous result to Proposition 4 above:

Proposition 5 *At equilibrium, when V is strictly concave,*

$$z > 0 \text{ if and only if } Ep < \overline{p}, \tag{20}$$

$$z < 0 \text{ if and only if } Ep > \overline{p}. \tag{21}$$

Proof. See the Appendix. ■

In other words, retailers will be net buyers in the spot market whenever expected spot-market price is below the price of long-term contracts, and *vice versa*. Clearly, retailers can only be net sellers in the spot market if they are over-contracted, that is, they have written long-term contracts for more than their retail sales, $\overline{z} > k$. Again, the result holds for all strictly concave V.

It follows from Propositions 4 and 5 that at equilibrium retailers and generators are on the same side of the spot market; that is, for all m and n, either $y_n > 0$ and $z_n < 0$ or $y_n < 0$ and $z_n > 0$. This perhaps somewhat counter-intuitive result follows from the assumption that all players face similar risk. Risk originates from spot-price volatility, caused by variations in net non-retail demand, Φ. Since risk affects retailers and generators symmetrically they all seek similar contractual positions in order to hedge. As discussed in the Conclusion, had we allowed for idiosyncratic risk we would typically not find all players on the same side of the spot market. However, even in this case market-wide risk would tend to move market participants in the same direction.

Note that, unless net non-retail demand Φ is identically zero, net spot-market supply of retailers and generators will be non-zero also. Therefore, expected spot-market price will in general differ from contract price; specifically, $Ep > \overline{p}$ if and only if $\Phi > 0$ at equilibrium; expected spot-market price and contract price are equal if and only if $\Phi(p, \theta) \equiv 0$.

Consider next the comparative statics of the contract market when the degree of generator involvement in retail is varied.

Lemma 4 *Generators reduce long-term contracts one-for-one against increased retail sales*

$$\frac{\partial \overline{y}}{\partial x} \equiv -1.$$

Proof. See the Appendix. ■

This result implies that increased vertical integration by generators functions as a transfer from their long-term contract portfolio into direct sales through vertical integration. Retail and contract sales are therefore perfect substitutes for the individual generator. This should not come as a surprise since, from the perspective of generators, the two sales channels have identical economic characteristics. This is a deliberate modelling choice on our behalf. In spite of this, vertical integration does have an impact on market outcomes, as we shall see below.

Retailers that are integrated with generators disappear as independent entities and their demand is directly served by the corresponding generators. In other words, when a generator takes over retailer sales $\triangle x$, the direct effect on demand in the spot market and the contract market is to reduce demand by $\triangle z$ and $\triangle \overline{z}$, respectively, where $\triangle z + \triangle \overline{z} = \triangle x$. The generator's supply in the spot market does not change, but its supply in the long-term contract market falls by $\triangle x$. Consequently, the direct effect of vertical integration is to reduce net demand by $\triangle \overline{z} - \triangle x$ in the contract market and by $\triangle z$ in the spot market. Adjustments then have to be made in order to achieve joint equilibrium in the two markets.

Clearly, if market participants were risk neutral, adjustment would leave prices in both spot and contract markets unchanged. Furthermore, output and consumption would not be affected. Indeed, one possible outcome in that case is that only the integrating generator undertakes adjustments in spot and contract trade so as to compensate for the demand reductions of the integrating retailer, leaving the rest of the market unchanged.

When market participants are risk averse, matters are different. To get a grip on how the equilibrium changes, we need to look in further detail into the partial effects of prices in spot and contract markets on generators and retailers.

Lemma 5 *In the contract market, generators' supply increases and retailers' demand falls with price, i.e.*

$$\begin{aligned} \frac{d\overline{y}}{d\overline{p}} &> 0 \\ \frac{d\overline{z}}{d\overline{p}} &< 0. \end{aligned}$$

Proof. See the Appendix. ■

It follows from the lemma that if, as a result of further vertical integration, net demand in the contract market is reduced - i.e. $\triangle \overline{z} - \triangle x > 0$ - equilibrium is restored by a decrease in market price; conversely, if increased integration raises net demand for long-term contracts - i.e. $\triangle \overline{z} - \triangle x < 0$ - the price of long-term contracts goes up.

Consider next implications for the spot market. The direct effect of vertical integration is to remove demand of $\triangle z$. Furthermore, changes in contract positions induce adjustments in the spot market. Referring to the result presented in Lemma 3, an increase in net contract demand of $\triangle\overline{z} - \triangle x$ induces a decrease in net demand in the spot market of $\gamma\,[\triangle\overline{z} - \triangle x]$, $0 < \gamma \leq 1$. Combining the direct effect of $\triangle z$ and the indirect effect of $\gamma\,[\triangle\overline{z} - \triangle x]$, we find that the total effect on the spot-market net demand is $-\triangle z + \gamma\,[\triangle\overline{z} - \triangle x] = -[1-\gamma]\,\triangle z$, which has the same sign as $-\triangle z$ but is smaller in magnitude. Consequently, from Lemma 2, we see that if vertical integration leads to a decrease in demand in the spot market - i.e. $\triangle z > 0$ - the effect is to induce a fall in the spot price; conversely, if integration increases net demand in the spot market - i.e. $\triangle z < 0$ - the spot price rises. In any case, the volume of trade in the spot market - measured by $|Y + Z|$, or, equivalently, $|\Phi|$ - is reduced.

Note that $\triangle\overline{z} - \triangle x = -\triangle z$, so $\triangle\overline{z} - \triangle x < 0$ if $\triangle z > 0$, and *vice versa*. It follows that prices in the spot market and contract market move in opposite directions as a result of increased vertical integration: if the market is initially in an equilibrium where retailers are buyers in the spot market, i.e. $z > 0$, spot price will fall following an increase in vertical integration, whereas contract price will rise. Furthermore, by Proposition 4 we initially must have $\overline{p} > Ep$ in this case, so the difference between spot price and contract price is increased following more vertical integration. Since generators set their production such that $c' = \overline{p}$ it follows that their output goes up and the difference between c' and Ep is increased. Vertical integration therefore affects output decisions and also exacerbates inefficiencies in allocation, manifested by the difference between marginal generation cost and expected spot price (which, in the absence of other market imperfections, reflects marginal willingness to pay associated with net non-retail demand).

If retailers are net sellers price effects in spot and contract markets are reversed. As before, allocative inefficiencies are aggravated by vertical integration. In this case, however, output falls.

We summarise these considerations in the following proposition.

Proposition 6 *When markets participants are risk-averse, the extent of vertical integration between generators and retailers affects market prices, net spot trade and production. Specifically, increased vertical integration implies greater difference between expected spot price on the one hand and marginal generation cost and long-term contract price on the other; allocative inefficiency is therefore exacerbated. Furthermore, output falls if generators are net sellers in the spot market and increases if they are net buyers. The volume of net trade in the spot market is unambiguously reduced.* ■

To understand why output decisions are affected we need to refer to the trade-off between profit and risk discussed in the previous section. When generators are net sellers in the spot market, expected spot price exceeds the contract price. Vertical integration leads to a price increase in the spot market and a price fall in the contract market. Contract adjustments therefore induce generators to shift supply from the contract market to the spot market, thereby increasing their exposure to risk. In order to mitigate this effect, generators reduce output. An alternative way to look at this is to note that when an independent retailer is vertically integrated with a generator net trade is removed from the spot market. Other market participants will have to make up for this imbalance, thereby increasing their risk exposure. This warrants an adjustment in the price level, driving a larger wedge between spot and contract prices.

5 Incentives for Vertical Integration

Now we consider the incentives of generators and retailers for vertically integrating, at Stage 1 of the game.

The maximum that a generator will be willing to pay for increased retail involvement dx, denoted dW, may be defined as follows:

$$E\left(U\left(\pi^{g}\left(x+dx\right)-dW\right)\right)=E\left(U\left(\pi^{g}\left(x\right)\right)\right), \tag{22}$$

where we have suppressed the quantities traded through long-term contracts and in the spot market. Similarly, the minimum payment a retailer would require in order to accept giving up sales of dk, denoted dA, may be defined by

$$E\left(V\left(\pi^{r}\left(k-dk\right)+dA\right)\right)=E\left(V\left(\pi^{r}\left(k\right)\right)\right). \tag{23}$$

Vertical integration is profitable when there exist a generator and an independent retailer for which $\frac{dW}{dx} > \frac{dA}{dk}$.

It follows from the Envelope Theorem that for generators $\frac{dW}{dx} = r - c'$ and for retailers $\frac{dA}{dk} = r - \overline{p}$. Since, in equilibrium, we have $c' = \overline{p}$ it follows that $\frac{dW}{dx} = \frac{dA}{dx}$ for all pairs of generators and retailers. In other words, the willingness-to-pay of a generator and the willingness-to-accept of a retailer will be identical at the margin. In particular, if $r > \overline{p}$ generators and retailers put the same positive value on increased retail sales. It follows that at the unique feasible transfer price market participants will be indifferent towards vertical integration

Even if individual firms are assumed to neglect market effects it is of interest to study overall effects of shifts in industry structure. If we take market effects into account, for generator n we find, by implicit differentiation of (22),

$$E\left(U'\left[\frac{\partial\pi^g}{\partial x}dx+\frac{\partial\pi^g}{\partial p}dp+\frac{\partial\pi^g}{\partial\overline{p}}d\overline{p}-dW\right]\right)=0. \tag{24}$$

Using the facts that $\frac{\partial\pi^g}{\partial x}=r-c'$, $\frac{\partial\pi^g}{\partial p}=y$ and $\frac{\partial\pi^g}{\partial\overline{p}}=\overline{y}$, we get

$$\begin{aligned} dW &= [r-c']\,dx+\overline{y}d\overline{p}+y\frac{E\left(U'dp\right)}{E\left(U'\right)} && (25)\\ &= [r-\overline{p}]\,dx+\overline{y}d\overline{p}+y\left[E\left(dp\right)+\frac{cov\left(U',dp\right)}{E\left(U'\right)}\right]. && (26)\end{aligned}$$

where we have used the observation that at equilibrium $c'\equiv\overline{p}$, and the equation (32) (see the Appendix) The first term after the second equality represents the gain from transferring sales from long-term wholesale contracts to retail. The second term is the price effect on contract sales. The last term is the risk-adjusted price effect on spot sales. The sign of risk adjustment $\frac{cov(U',dp)}{E(U')}$ depends on the generator's net position in the spot market, as well as on how vertical integration affects spot-price variability.

For net sellers, profits are increasing, and hence marginal utilities are decreasing, in spot price. The reverse holds for net buyers. The price change resulting from increased vertical integration is a stochastic variable which will vary with realised net non-retail demand, or spot price. In the same manner as in the proof of Lemma 2, it is easily seen that dp will vary positively or negatively with p depending on whether vertical integration increases or decreases price variability. If net non-retail demand satisfies inequality (15), price variability goes up; if the reverse inequality holds, price variability falls.

Therefore, risk adjustment is negative if generators are sellers in the spot market and the inequality (15) holds; then marginal utility is low and price changes are large for high realisations of the spot price, and *vice versa*. Risk adjustment is negative also if generators are buyers in the spot market and the inequality in (15) is reversed. Conversely, risk adjustment is positive if either generators are buyers and (15) holds or if generators are sellers and the reverse inequality holds.

It follows that $y\frac{cov(U',dp)}{E(U')}$, the net contribution to dW, is negative or positive depending on whether the inequality (15) or the reverse inequality holds, respectively.

Similarly, for a retailer, we find from (23) that

$$E\left(V'\left[-\frac{\partial\pi^r}{\partial k}dk+\frac{\partial\pi^r}{\partial p}dp+\frac{\partial\pi^r}{\partial\overline{p}}d\overline{p}+dA\right]\right)=0, \tag{27}$$

from which it follows, using the facts that $\frac{\partial \pi^t}{\partial k} = r - \overline{p}$, $\frac{\partial \pi^r}{\partial p} = -z$ and $\frac{\partial \pi^r}{\partial \overline{p}} = -\overline{z}$, that

$$\begin{aligned} dA &= [r - \overline{p}]\, dk + \overline{z} d\overline{p} + z \frac{E(V' dp)}{E(V')} && (28) \\ &= [r - \overline{p}]\, dk + \overline{z} d\overline{p} + z \left[E(dp) + \frac{cov(V', dp)}{E(V')} \right], && (29) \end{aligned}$$

which has an analogous interpretation to the corresponding expression for generators.

Setting $dk = dx$, for a given generator-retailer pair, we find that the net surplus from vertical integration becomes

$$\begin{aligned} dW - dA &= [y - z]\, E(dp) + [\overline{y} - \overline{z}]\, d\overline{p} \\ &\quad + y \frac{cov(U', dp)}{E(U')} - z \frac{cov(V', dp)}{E(V')} \end{aligned}$$

The first term on the right-hand side represents the spot-price level effect. From the previous section, we know that $sign(y) = sign(-z)$ and that, as a result of increased vertical integration, $dp > 0$ if $y > 0$ and $dp < 0$ if $y < 0$. The spot-price level effect is therefore always positive. In other words, when generators and retailers are net sellers in the spot market, spot price goes up as a result of increased vertical integration, to the benefit of both parties; if they are net buyers, they gain from the resulting price fall.

The second term represents the effect of the resulting change of the contract price. In general, this term may be positive or negative depending on the net contract position of the generator and retailer involved, as well as the direction of the price change. Since, at equilibrium in the contract market, $\sum_n \overline{y}_n - \sum_m \overline{z}_m \equiv 0$ it follows that there must exist generator-retailer pairs such that $\overline{y} - \overline{z} \geq 0$ and others where $\overline{y} - \overline{z} \leq 0$. Hence, there always exists at least one pair for which the contract-price effect is positive.

The two last terms represent the effect of change in spot-market risk. These are positive or negative depending upon how vertical integration affects spot-price variability. Note, however, that, since $sign(y) \equiv sign(-z)$ these terms have the same sign.

It follows that, taking market effects into account, vertical integration is always profitable for some generator and retailer if price variability is reduced. The same is true if the effect on price variability is sufficiently small in magnitude, independent of its sign; for instance, when net non-retail demand is linear in price, price variability does not depend on vertical integration. Only if price variability increases sufficiently will vertical integration be unprofitable.

If we consider the industry as a whole, the net surplus from vertical integration becomes

$$\sum_n dW_n - \sum_m dA_m = \Phi E(dp) + \sum_n y_n \frac{cov(U_n', dp)}{E(U_n')} - \sum_m z_m \frac{cov(V_m', dp)}{E(V_m')}.$$

The overall effect is therefore composed of the positive spot-price level effect and the effect of changes in spot-market risk, which may be positive or negative. Unless the risk effect is negative and substantial in absolute value the industry as a whole will be in favour of vertical integration.

6 Retail prices linked to spot prices

In this section we consider another possible assumption about retail prices, namely that retail prices are linked to spot market prices. An example of such a price link is found in the Danish electricity market, where regulation requires incumbent suppliers to offer households and other small consumers a default contract in which price is proportional to the spot price. Of course, retail prices that are closely linked to spot prices may also be the result of competition; in Norway, for example, spot-related pricing is common in retail.[9]

Roughly speaking, when retail prices are proportional to spot price - in particular, if $r = p$ - we can derive corresponding results to those above. More formally, the following result is easily established.

Proposition 7 *Suppose $r = p$. Then, at equilibrium, when U and V are strictly*

[9] In Norway, there are in effect three types of retail contracts: "spot-price contracts", in which retail price equals the spot price plus a fixed or percentage mark up; "variable-price contracts", in which price may be adjusted bi-weekly, but in practice follows the spot price; and "fixed-price contracts", in which prices are set for a longer period, typically a year. In the household segment, around 80 % of retail sales are under the former two types of contracts; the corresponding figures are approx. 70 % for service industries and approx. 40 % for manufacturing industries (von der Fehr, Amundsen and Bergman, 2005).

concave,

$$\begin{aligned} c'(q) &= \overline{p} < Ep \ \ \textit{if and only if} \ \ y + x > 0, \\ c'(q) &= \overline{p} > Ep \ \ \textit{if and only if} \ \ x + y < 0, \\ -1 &< \frac{\partial y}{\partial \overline{y}} < 0, \quad \frac{\partial y}{\partial x} \equiv -1, \quad \frac{\partial \overline{y}}{\partial x} \equiv 0 \\ \overline{z} &> 0 \ \ \textit{if and only if} \ \ \overline{p} < Ep, \\ \overline{z} &< 0 \ \ \textit{if and only if} \ \ \overline{p} > Ep. \end{aligned}$$

Proof. See the Appendix. ■

Note that in this case generators' exposure to spot-price risk depends on their combined position in spot and retail markets. Hence, net sellers in these two markets combined underproduce; that is, they set their production such that marginal cost is lower than expected spot price. The reverse holds for net buyers. Furthermore, generators will be net sellers if and only if expected spot price is higher than the contract price, and *vice versa*.

For generators, when retail price is linked to spot price retail sales and spot sales are perfect substitutes from a risk perspective, and the equivalence between long-term contracts and vertical integration that we observed in the case of fixed retail prices no longer holds. It follows that increased vertical integration is fully offset by reduced spot sales with no direct effect on contract position. However, since integrating into retailing does not provide any hedge against spot price risk there is a hedging incentive to sell on contract. As before, changes in contract positions are partly offset by opposite adjustment in spot trade.

For retailers, the spot market now provides a perfect hedge and they have no hedging motive for entering the market for long-term contracts. Consequently, retailers buy in the contract market if and only if the price there is below expected spot price.

We have the following result:

Proposition 8 *When retail price is linked to spot price, price on long-term contracts must be below expected spot price at equilibrium, i.e.*

$$\overline{p} < Ep.$$

Proof. See the Appendix. ■

The result follows from the following observations: Suppose that the contract price is above expected spot price, i.e. $\overline{p} > Ep$. As explained above, retailers would then sell in the contract market. Moreover, generators would be net buyers in the combined spot and retail markets and hence would be sellers in the contract market also. Hence, all participants in the contract market would be sellers, which is clearly impossible.[10]

Turning to the impact of vertical integration, it immediately follows that when a generator takes over retailer sales $\triangle x$, the direct effect of vertical integration is to reduce net demand by $\triangle z - \triangle x$ in the spot market and by $\triangle \overline{z}$ in the contract market. As before, the direct effect on retailer demand in the spot market and contract markets is to reduce demand by $\triangle z$ and $\triangle \overline{z}$, respectively, where $\triangle z + \triangle \overline{z} = \triangle x$. The generator's supply in the long-term market does not change, but its supply in the spot market falls by $\triangle x$.

Since $Ep > \overline{p}$, by Proposition 7 above, retailers are buyers in the contract market, so $\triangle \overline{z} > 0$ and $\triangle z - \triangle x = -\triangle \overline{z} < 0$. Therefore, the direct effect of vertical integration is to reduce net demand in the contract market and increase it in the spot market. It follows that the contract price must fall, while the spot price must rise, increasing the difference between expected spot price and price in the contract market.

The fact that vertical integration affects wholesale prices in the same manner both when retail prices are fixed and when retail prices vary with spot prices is due to different market behaviour of retailers in the two cases. When retail prices are fixed, contract purchases provide a perfect hedge for retailers and hence they only buy in the risky spot market whenever prices there are on average lower than in the contract market. When retail prices vary with spot prices, on the other hand, the spot market provides hedging and hence purchases in the contract market only occur when prices there are sufficiently low. It follows that in both cases the effect of vertical integration will be to remove demand from the market where price is lower and add demand in the market where the price is higher. Take the case when expected spot price exceeds contract price as an example. When retail prices are fixed, retailers will be buying more electricity in the contract market than they need to serve their retail demand; hence, following vertical integration, the reduction in the integrating generator's contract supply is more than compensated for by the fall in the integrating retailer's contract demand and so net demand for contracts is reduced. When retail prices vary, retailers are buyers in the contract market, and, since generator contract supply is not affected by vertical integration, net demand falls in this case also.

We conclude that the qualitative effects of vertical integration on market perfor-

[10]Note that this argument relies on the assumption of no idiosyncratic risk, cf. the discussion after Proposition 5.

mance are independent of how retail prices are determined.

Turning to the incentives for vertical integration, we find, using the same notation as before, that $\frac{dW}{dx} = \frac{dA}{dk} = 0$; in other words, both generators and retailers are indifferent to increased retail sales. This is intuitive, since retail prices are identical to spot price. Taking market effects into account, we again find that vertical integration has a positive spot-price-level effect, a positive contract-price-level effect for some generator-retailer pair, and an ambiguous spot-market risk effect; in particular,

$$\begin{aligned} dW - dA &= [x + y + \overline{z}] E(dp) + [\overline{y} - \overline{z}] d\overline{p} \\ &\quad + [x + y] \frac{cov(U', dp)}{E(U')} + \overline{z} \frac{cov(V', dp)}{E(V')}. \end{aligned}$$

For the industry as a whole we have

$$\sum_n dW_n - \sum_m dA_m = QE(dp) + \sum_n [x_n + y_n] \frac{cov(U'_n, dp)}{E(U'_n)} + \sum_m \overline{z}_m \frac{cov(V'_m, dp)}{E(V'_m)}.$$

where $Q = \sum_n q_n$ denotes total generation. Therefore, also in this case the industry will be in favor of vertical integration, unless the risk effect is negative and substantial in absolute value .

7 Market power

We now turn to the issue of market power. We concentrate on the case in which retail prices are fixed and only generators have market power; specifically, we assume that generators behave as Cournot quantity setters on wholesale markets.

Consider first the spot market. Above, we demonstrated that, in the case of price-taking behaviour, risk-averse generators underproduce when they are net sellers in the spot market, and *vice versa*. Intuitively, we would expect market power to enhance this effect; in particular, a seller, in order to drive up price, would want to restrict supply; and, conversely, a buyer, in order to drive down price, would want to restrict purchases. This turns out to be true, under a further technical condition on the net-retail demand function to ensure that generator profits are increasing in the demand shock (see the Appendix for a discussion of this condition):

Proposition 9 *Suppose* $\frac{\alpha\varepsilon_\theta}{\varepsilon} > -1$, *where* $\varepsilon = -\frac{p\frac{\partial\Phi}{\partial p}}{\Phi}$ *is the elasticity of* Φ *(net retail demand),* $\varepsilon_\theta = -\frac{\frac{\partial^2\Phi}{\partial p\partial\theta}}{\frac{\partial\Phi}{\partial\theta}/p}$ *is the elasticity of* $\frac{\partial\Phi}{\partial\theta}$ *(net non-retail demand, marginal with* *respect to the demand shock θ) and $\alpha = \frac{y}{\Phi}$ is generator spot-market share. At the Cournot Nash equilibrium we then have*

$$c'(q) < E\left(p + \frac{dp}{dy}y\right) < Ep \quad \textit{if} \quad y > 0, \tag{30}$$

$$c'(q) > E\left(p + \frac{dp}{dy}y\right) > Ep \quad \textit{if} \quad y < 0 \tag{31}$$

Proof. See the Appendix. ■

A risk-neutral generator with market power would equate marginal cost to (expected) marginal revenue, which falls below (expected) market price if the generator is a net seller in the spot market. A risk-averse generator would sell even less on the spot-market, thereby increasing the discrepancy between marginal cost and (expected) spot price. Similarly, a risk-averse generator who buys from the spot market would reduce purchases both to limit exposure to risk and to drive down price. Since generators tend to be sellers in the spot market when (expected) spot price exceeds the contract price, and *vice versa*, *ceteris paribus* the incentives arising from the spot market tend to drive spot price further away from contract price.

Matters are not so straightforward in the market for long-term contracts. As seen above, generators that take prices as given equate marginal cost to the (non-stochastic) contract price, irrespective of their attitude towards risk. A risk-neutral generator would equate marginal cost to marginal revenue, which exceeds or falls below the contract price depending upon whether the generator is a seller or a buyer in the contract market. If such a generator has a different trading position in the contract market relative to that in the spot market exercising market power would drive spot and contract prices apart; for example, if the generator is a seller in the spot market and a buyer in the contract market, the incentive is to both drive up the spot price and drive down the contract price. However, if the generator has the same net trading position on both markets, the incentive arising from market power is to drive prices in the same direction: for example, if the generator is a net seller on both markets, the incentive is to drive up both prices, and whether they will be brought closer together depends on the finer details of the model.

The analysis is further complicated when we take risk aversion into account. The first-order condition for maximisation of expected utility at Stage 2 of the game may be written

$$E\left(U'\frac{\partial\pi}{\partial\overline{y}}\right) + E\left(U'\frac{\partial\pi}{\partial y}\frac{dy}{d\overline{y}}\right) + \sum_{i\neq n} E\left(U'\frac{\partial\pi}{\partial y_i}\frac{dy_i}{d\overline{y}}\right) = 0.$$

The first term represents the marginal gain from greater contract sales, the second term represents the marginal gain from the generator's own spot-market adjustments (and is equal to zero by the Envelope Theorem), while the third term represents the strategic effect on competitors' spot-market behaviour.

Exploiting the facts that $\frac{\partial\pi}{\partial\overline{y}} = \overline{p} + \frac{d\overline{p}}{d\overline{y}}\overline{y} - c'$ is deterministic and $\frac{\partial\pi}{\partial y_i} = \frac{dp}{dy_i}y$ is independent of i, the first-order condition may alternatively be written

$$\overline{p} + \frac{d\overline{p}}{d\overline{y}}\overline{y} - c' + \frac{E\left(U'\frac{dp}{dy_i}\right)}{E\left(U'\right)}y\sum_{i\neq n}\frac{dy_i}{d\overline{y}} = 0.$$

Here the first two terms represent marginal revenue from increased contract sales, while the second term is marginal cost of adjusting output to allow for such an increase in sales. The last term represents the effect on spot-market risk exposure induced by the effect of the change in contract sales on competitors' spot market supplies; this term may be positive or negative depending on the characteristics of non-retail demand Φ as well as whether the generator is a net seller or buyer in the spot market (i.e. the sign of y). It follows that we cannot unambiguously determine the effect of market power on behaviour in the contract market and hence on how market power affects the relation between spot and contract prices.

8 Concluding remarks

The analysis in this paper is motivated by concerns expressed by regulatory authorities regarding the effects of vertical relations on market liquidity, price volatility and risk. We have suggested a modelling framework for analysing these issues. In this framework risk matters; compared to the benchmark case of risk neutrality, there will be less trade in the spot market when agents are risk averse; there will also be a wedge between (expected) spot and contract prices.

We have demonstrated that vertical integration will be offset, but only partly, by trade adjustments in contract and spot markets. The extent of vertical integration therefore affects market performance; a higher degree of vertical integration leads to reduced net trade ("liquidity") in the spot market; it may also lead to increased spot-price variability, although that depends on the characteristics of net non-retail demand. Vertical integration between generators and retailers in effect removes retailers - who generally operate in both spot and contract markets - from the wholesale market. Even if integrating generators compensate one-for-one by adjusting their positions in relevant markets, the overall effect will be to shift supply and demand from one market to another. These shifts affect the level of prices as well as their volatility. As a consequence, agents adjust market positions in order to compensate for changes in risk, thereby influencing volumes of trade as well prices. We have seen that results are qualitatively similar independently of whether retail prices are fixed or variable (i.e. linked to the wholesale spot price).

While the model is admittedly highly simplified and stylised, we do not believe the fundamental nature of our results depend essentially on our modelling choices. For example, the assumption of simple quantity bids (in combination with elastic demand) is natural given our desire to abstract from market power issues. If we were instead to assume more realistic spot-market (supply-function) bidding schedules[11] we would find that, under perfect competition, risk-neutral agents would submit bid functions identical to marginal cost curves, whereas with risk aversion supply functions would lie above or below marginal-cost curves depending on agents' net position in the spot market. Otherwise, results would be similar, with the exception that in this case price variability would be driven by the shape of supply functions (which would depend on the shape of underlying costs), as well as demand. More generally, the effect of vertical integration on spot-price variability depends on both demand and supply technology and hence cannot be determined without detailed knowledge about functional forms.

Similarly, our assumption about origin of uncertainty greatly simplifies the analysis without seemingly affecting the essence of results. In particular, the assumption that uncertainty originates from net non-retail demand in the spot market means that generators and retailers face only extraneous risk, via the spot price. In practice, idiosyncratic risk may be important also. In general there could be several sources of shocks in the model: net non-retail demand in the spot market, Φ may be stochastic; demand at each retail outlet, k_m, may also vary randomly; and, finally, available capacity and cost at each generation plant may be uncertain. Our earlier work on related issues suggests that the main insights of the present setup would survive a generalisation to such types of risk (see Baldursson and von der Fehr, 2004). Specifically, even in the presence of idiosyncratic risk market-wide risk would tend to move all agents in the same direction, affecting both market price and trade.

Finally, our modelling framework is "continuous" in the sense that all demand and

[11] For two alternative approaches to modelling spot-market interaction see Green and Newbery (1992) and von der Fehr and Harbord (1994); for a discussion see Fabra, von der Fehr and Harbord (2002, 2006).

cost technologies are smooth and there are no fixed costs of adjustment or transactions. The implication is that agents participate in all markets and that they undertake continuous adjustments as a consequence of changes in underlying structure, such as increased vertical integration. In practice, markets are characterised by more discrete behaviour, such as jumps in prices due to the presence of bid-ask spreads, and shifts in market activity caused by firms' decisions to enter or withdraw from a particular market. Our model does not capture such features. For example, we show that vertical integration may lead to greater spot-price volatility; this - in a world of discreteness - could lead to greater bid-ask spreads, further amplification of volatility and ultimately withdrawal of market participants. We would, however, expect the effects identified in our analysis to be present in a richer model.

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A Proofs

A.1 Proposition 1

Note that if two random variables ς and υ are non-negatively correlated then

$$\begin{aligned} E(\varsigma\upsilon) &= E(\varsigma)E(\upsilon) + Cov(\varsigma,\upsilon) & (32)\\ &\geq E(\varsigma)E(\upsilon) & (33) \end{aligned}$$

with strict inequality if $Cov(\varsigma,\upsilon) > 0$. Furthermore, it is easy to show that if f and g are strictly increasing functions, then $f(\varsigma)$ and $g(\varsigma)$ are strictly positively correlated, unless ς is deterministic. We then have $E(f(\varsigma)g(\varsigma)) > E(f(\varsigma))E(g(\varsigma))$.

Consider the random variables $\varsigma = U'(\pi^g)$ and $\upsilon = p - c'(q)$, which are both functions of the driving random variable, θ. From the equilibrium condition (5) and the properties of the demand function (6a,6b), we have

$$\frac{dp}{d\theta} = -\frac{\frac{\partial\Phi}{\partial\theta}}{\frac{\partial\Phi}{\partial p}} > 0. \quad (34)$$

So, the spot price is increasing in θ. Then, if $y > 0$, by (2), profit is increasing in θ, and, since U is concave, marginal utility $\varsigma = U'(\pi^g)$ is decreasing in θ. Furthermore, marginal profit, $\upsilon = p - c'(q)$, is increasing in θ. It follows from (33) - applied to negatively correlated random variables - that

$$E\left(U'\left(\pi\right)\right)E\left(p-c'\left(q\right)\right)>E\left(U'\left(\pi\right)\left[p-c'\left(q\right)\right]\right). \tag{35}$$

The conclusion (10) now follows from the first-order condition (8) and the fact that $U'\left(\pi\right)>0$.

Conversely, when $y<0$, profit is decreasing and $U'\left(\pi\right)$ is increasing in θ. Marginal profit is increasing in θ as before. Now the inequality (11) follows from an application of (33) to $U'\left(\pi\right)$ and marginal profit. ■

A.2 Lemma 1

The proof proceeds by differentiation of the first-order condition (8). The calculation is exactly the same in both cases, i.e. for x and $\overline{y}$, so we only do the latter. To save space we omit arguments of functions in the resulting equation:

$$E\left(U''\left\{\overline{p}+p\frac{\partial y}{\partial\overline{y}}-c'\left[1+\frac{\partial y}{\partial\overline{y}}\right]\right\}\left[p-c'\right]\right)-E\left(U'c''\left[1+\frac{\partial y}{\partial\overline{y}}\right]\right)=0. \tag{36}$$

Collecting terms we get

$$-\frac{\partial y}{\partial\overline{y}}\left[E\left(-U''\left[p-c'\right]^2\right)+E\left(U'c''\right)\right]+\left[\overline{p}-c'\right]E\left(U''\left[p-c'\right]\right)-E\left(U'c''\right)=0. \tag{37}$$

Note that the second term on the left-hand side of this equation is identically equal to zero by the CARA assumption and the first-order condition (8). We therefore have

$$\frac{\partial y}{\partial\overline{y}}=-\frac{E\left(U'c''\right)}{E\left(U'c''\right)+E\left(-U''\left[p-c'\right]^2\right)}, \tag{38}$$

and, since U is increasing and concave so $E\left(U'c''\right)>0$ and $E\left(-U''\left[p-c'\right]^2\right)\geq 0$, it follows that $-1\leq\frac{\partial y}{\partial\overline{y}}<0$. If U is strictly concave then $E\left(-U''\left[p-c'\right]^2\right)>0$ and $\frac{\partial y}{\partial\overline{y}}>-1$. ■

A.3 Lemma 2

From the spot-market equilibrium condition (5) we get

$$\frac{dp}{ds}=\left[\frac{\partial\Phi}{\partial p}\right]^{-1}<0 \tag{39}$$

which proves that spot prices fall when net supply increases. Differentiating this equation with respect to θ and reversing the order of differentiation we have

$$\frac{d}{ds}\frac{dp}{d\theta}=-\left[\frac{\partial^2\Phi}{\partial p\partial\theta}\frac{\partial\Phi}{\partial p}-\frac{\partial^2\Phi}{\partial p^2}\frac{\partial\Phi}{\partial\theta}\right]\left[\frac{\partial\Phi}{\partial p}\right]^{-3}. \tag{40}$$

Note that $\frac{dp}{d\theta}$ is the extent to which fluctuations in the demand shock translate into variability of prices. The left-hand side of the equation therefore indicates the effect of an increase in net supply on price variability. Furthermore, since $\frac{\partial\Phi}{\partial p} < 0$ the right-hand side of the equation is positive if (and only if) (13) holds. ■

A.4 Lemma 3

Suppose the change in net supply to the contract market is $d\overline{s} > 0$. The induced effect on net supply in the spot market is

$$\begin{aligned} ds &= \sum_{n=1}^{N} dy_n - \sum_{m\in M_0} dz_m \\ &= \sum_{n=1}^{N} \frac{\partial y_n}{\partial \overline{y}_n} d\overline{y}_n - \sum_{m\in M_0} \frac{\partial z_m}{\partial \overline{z}_m} d\overline{z}_m \\ &\geq -\sum_{n\neq 1} d\overline{y}_n + \sum_{m\in M_0} d\overline{z}_m \qquad (41) \\ &= -d\overline{s} \end{aligned}$$

For the former inequality we rely on the inequalities $d\overline{y}_n < 0$, for all $n > 1$, and the observation that $\frac{\partial y_n}{\partial \overline{y}_n} \geq -1$ by Lemma 1 and $\frac{\partial z_m}{\partial \overline{z}_m} = -1$ by (3). For the last equality we use the definition of $d\overline{s}$. Since $\frac{\partial y_n}{\partial \overline{y}_n} < 0$, it is also easily seen that $ds < \sum_{m\in M_0} d\overline{z}_m < 0$. It follows that $-d\overline{s} \leq ds < 0$. An analogous derivation shows that if $d\overline{s} < 0$ then $-d\overline{s} \geq ds > 0$. The inequalities (14) follow. ■

A.5 Proposition 3

Note that y^* maximises $EU\left(\pi^g\left(x, \overline{y}, y\right)\right)$ for given x and $\overline{y}$. We then have by the Envelope Theorem

$$\frac{dEU\left(\pi^{g*}\right)}{d\overline{y}} = E\left(U'\left(\pi^{g*}\right)\left[\overline{p} - c'\left(x + \overline{y} + y^*\right)\right]\right), \qquad (42)$$

where we have written $\pi^{g*} = \pi^g\left(x, \overline{y}, y^*\right)$. Since $\overline{p}$, x, $\overline{y}$ and y^* are all deterministic (i.e. determined prior to the realization of Φ), the first-order condition for maximisation of $EU\left(\pi^{g*}\right)$ reduces to (16). ■

A.6 Proposition 5

Note that $\pi_m^r = rk - p\left[k - \overline{z}\right] - \overline{pz}$ is decreasing in p if and only if $z = k - \overline{z} > 0$, i.e. if and only if the retailer is a buyer in the spot market. Hence, $V'\left(\pi_m^r\right)$ is increasing in

p if and only if $k - \overline{z} > 0$. From the first-order condition we get for strictly concave V

$$\begin{aligned} 0 &= E\left(V'\left(\pi_m^r\right)[p-\overline{p}]\right) && (43) \\ &= EV'\left(\pi_m^r\right)E\left(p-\overline{p}\right)+Cov\left(V',p\right) && (44) \\ &> EV'\left(\pi_m^r\right)E\left(p-\overline{p}\right) && (45) \end{aligned}$$

Since $V' > 0$ this implies that we must have $Ep < \overline{p}$. On the other hand, if $k - \overline{z} < 0$, so retailers are net sellers in the spot market, $V'\left(\pi_m^r\right)$ is increasing in p so the reverse inequality obtains, $Ep > \overline{p}$. ■

A.7 Lemma 4

We differentiate generators' first-order condition (16) for maximisation of expected utility at the contract stage with respect to retail sales x and get

$$c''\left[1+\frac{\partial y}{\partial x}\right]\left[1+\frac{\partial \overline{y}}{\partial x}\right]=0. \tag{46}$$

Since $c'' > 0$ and, by Lemma 1, $1+\frac{\partial y}{\partial x} > 1$ the result follows. ■

A.8 Lemma 5

From (16), applying Lemma 1, we have

$$\frac{d\overline{y}}{d\overline{p}}=\left\{c''\left[1+\frac{\partial y^*}{\partial \overline{y}}\right]\right\}^{-1}>0, \tag{47}$$

Furthermore, differentiationg the first-order condition (19) with respect to $\overline{p}$, we find

$$\frac{d\overline{z}}{d\overline{p}}=\frac{\overline{z}E\left(V''\left[p-\overline{p}\right]\right)+E\left(V'\right)}{E\left(V''\left[p-\overline{p}\right]^2\right)}<0. \tag{48}$$

■

A.9 Proposition 7

The proof of the lemma follows, *mutatis mutandis,* the presentation in the case of fixed retail prices step-by-step and we only give a rough outline here.

The first two sets of inequalities are proven exactly as in Lemmata 1 and 3 using the first-order conditions for generators.

Proving that $-1 < \frac{\partial y}{\partial \overline{y}} < 0$ is done exactly as in the proof of Lemma 1. Proving that $\frac{\partial y}{\partial x} \equiv -1$ follows the same pattern, but the term $\left[\overline{p}-c'\right]E\left(U''\left[p-c'\right]\right)$ in (37) is replaced by $E\left(U''\left[p-c'\right]^2\right)$ (note that this result does not require the CARA assumption on U). The last identity follows from differentiation of the first-order condition for generators (16), which yields

$$\frac{\partial \overline{y}}{\partial x}\left[1+\frac{\partial y}{\partial \overline{y}}\right]=0. \tag{49}$$

Since $\frac{\partial y}{\partial \overline{y}} > -1$ this implies that $\frac{\partial \overline{y}}{\partial x} \equiv 0$.

The last set of inequalities is proved as in the proof of Proposition 5. Note that here retailer profits reduce to $\pi_m = rk - pz - \overline{pz} = [p - \overline{p}]\,\overline{z}$, implying that the relation between profit and spot price depends on whether the retailer is a buyer or seller in the contract market. ■

A.10 Proposition 8

Suppose $\overline{p} > Ep$. Then, from Lemma 7, it follows that $\overline{z} < 0$ for all retailers. Furthermore, for all generators $x + y < 0$, implying that $\overline{y} = q - x - y > 0$. Therefore, $\sum_{n=1}^{N} \overline{y}_n - \sum_{m\in M_0} \overline{z}_m > 0$, which contradicts the equilibrium condition (7). ■

A.11 Proposition 9

The first-order condition for maximisation of expected utility of a generator at Stage 3 is (we omit the subscript n)

$$E\left(U'(\pi)\left[p+\frac{dp}{dy}y-c'(q)\right]\right)=E\left(U'(\pi)\left[p\left[1-\frac{\alpha}{\varepsilon}\right]-c'(q)\right]\right)=0, \tag{50}$$

where $\varepsilon = -\frac{p\frac{\partial \Phi}{\partial p}}{\Phi}$ (we have made use of (5) to show $\frac{dp}{dy} = \left[\frac{\partial \Phi}{\partial p}\right]^{-1}$) and $\alpha = \frac{y}{\Phi}$. The second-order condition for maximisation of expected utility is

$$E\left(U''\left[p+\frac{dp}{dy}y-c'\right]^2\right)+E\left(U'\left[2\frac{dp}{dy}+y\frac{d^2p}{dy^2}-c''\right]\right)<0, \tag{51}$$

where we have omitted the arguments of the functions involved. The first term is unambiguously negative. In the second term, $U' > 0$, $\frac{dp}{dy} < 0$ and $c'' > 0$ so it is only the term $U'y\frac{dp}{dy}$ that may be non-negative. However, we assume throughout that this second-order condition holds.

We first consider the case in which generators are not risk averse, i.e. they maximise expected profit. In this case the first-order condition for maximisation of profit becomes

$$E\left(p+\frac{dp}{dy}y-c'(q)\right)=E\left(p\left[1-\frac{\alpha}{\varepsilon}\right]-c'(q)\right)=0. \tag{52}$$

Before going further it is useful to establish some properties of the spot-market equilibrium. Suppose prices and quantities in the retail and long-term contract markets are given for all generators. Then, in spot-market equilibrium, we have

$$\frac{dp}{d\theta} = -\frac{\frac{\partial\Phi}{\partial\theta}}{\frac{\partial\Phi}{\partial p}} > 0 \tag{53}$$

$$\frac{dp}{dy_n} = \left[\frac{\partial\Phi}{\partial p}\right]^{-1} < 0 \tag{54}$$

$$\frac{d^2\pi_n^g}{d\theta dy_n} = -\frac{\frac{\partial^2\Phi}{\partial p\partial\theta}}{\left(\frac{\partial\Phi}{\partial p}\right)^2}y + \frac{dp}{d\theta}. \tag{55}$$

The result follows from differentiation of the condition for spot-market equilibrium (5), the expression for profit of individual generators (2) and the properties of the demand function (6).

The following result, which parallels the results of eg. Bushnell, Mansur and Saravia (2006) in that it is driven purely by market power, then follows directly from (52) and (54). Suppose prices and quantities in the retail and long-term contract markets are given for all generators. Also assume generators are risk-neutral. For a single generator who submits (Cournot) bids into the spot market we then have

$$c'(q) = E\left(p + \frac{dp}{dy}y\right)\begin{cases} < Ep & \text{if} \quad y > 0 \\ > Ep & \text{if} \quad y < 0 \end{cases} \tag{56}$$

We have now established the key result under the condition that either generators are risk-averse, but do not exert market power, or that they are risk-neutral, but do exert market power. One would then intuitively expect that it is a relatively easy matter to show that when the two - i.e. risk-aversion and market power - are combined, each effects strengthens the other. This turns out to be true under a further technical condition on the demand function. Before stating the result we need to define the price elasticity of $\frac{\partial\Phi}{\partial\theta}$ (net non-retail demand, marginal with respect to the demand shock θ):

$$\varepsilon_\theta = -\frac{\frac{\partial^2\Phi}{\partial p\partial\theta}}{\frac{\partial\Phi}{\partial\theta}/p}. \tag{57}$$

It should be noted at this point that the proof of Proposition 1 hinges on the monotonicity of $U'(\pi)$ and marginal profit $\frac{d\pi}{dy}$ with respect to the aggregate demand shock θ. It turns out that marginal profit of generator n is increasing in θ if and only if

$$\frac{\alpha_n \varepsilon_\theta}{\varepsilon} > -1, \tag{58}$$

where, as before, $\varepsilon = -\frac{p\frac{\partial \Phi}{\partial p}}{\Phi}$ and $\alpha_n = \frac{y_n}{\Phi}$.

The remained of the proof proceeds in a fashion similar to the proof of Proposition 1. As before, $U'(\pi)$ is decreasing in θ whenever $y > 0$ and increasing in θ whenever $y < 0$. The condition (58) guarantees that marginal profit $p + \frac{dp}{dy}y - c'(q)$ is increasing in θ, c.f. (55). These properties, along with (**??**) and the first-order condition (50), suffice to establish the former inequalities in (30) and (31). The latter inequalities follow directly from (54).

We finally consider some parametrised examples:

1. The proposition will hold for a generator whose net market share α is small enough. In particular, if $\alpha = 0$ Proposition 9 holds unconditionally.

2. With a linear demand function of the form $\Phi(p, \theta) = A\theta - Bp$, where A and B are positive constants, it is immediate that (58) holds. Proposition 9 therefore holds unconditionally in this case and in general whenever the cross-derivative $\frac{\partial^2 \Phi}{\partial p \partial \theta}$ is equal to zero.

3. With a constant price elasticity demand function of the form $\Phi(p, \theta) = A\theta p^{-\varepsilon}$ we get $\alpha\varepsilon_\theta/\varepsilon \equiv \alpha$ so (58) boils down to the condition $\alpha > -1$, i.e. that the generator in question is not a net buyer of more than all the price elastic demand Φ. In this case net spot demand Φ is of course always positive. ■

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