Are Risky Workers More Valuable to Firms?

J Michael Orszag and Gylfi Zoega
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A worker’s performance may vary over time for reasons that have nothing to do with his inherent abilities, motivation, background and education. For example, over time the nature of a firm’s business may change to reduce the degree of match between the human capital of the worker and the needs of the firm’s business. When firms make hiring decisions, which are costly to implement or reverse, the uncertain human capital productivity has significant practical implications. Conventional wisdom is that workers whose future productivity is more risky should be paid less than those whose perceived future productivity is less risky. This paper shows that in many institutional contexts the conventional wisdom is incorrect: risky workers should be paid more rather than less because those in risky segments are that much harder to replace. But on the other hand, we find that higher wages among those whose value to firms is more uncertain is also matched by a higher degree of unemployment among these groups.

**Keywords:** Uncertainty, wage setting, hiring, employment.

**JEL:** J230, J240
1. Introduction

A worker’s performance may vary over time for reasons that have nothing to do with his inherent abilities, motivation, background and education. While firms adopt various procedures to reduce the level of uncertainty involved in hiring, it is clear that they can never predict a worker’s added value to an organisation with any degree of certainty.1 The treacherous seas of personal relationships, the good fortune of having a generous mentor and the personal chemistry one has with colleagues and friends can all affect performance on the job. Shifts in the market and the specific skill needs of the firm also play an important role. In the extreme, a marriage breakdown and health problems should have a big impact on one’s performance. Less apparent factors also count, for example a supportive and understanding spouse can bolster productivity and personal growth. While asymmetric information about inherent worker abilities can generate involuntary unemployment, as shown by Weiss (1990), in this paper we focus not on differences in inherent abilities but on the innumerable events that affect a worker’s productivity over his or her lifespan and derive implications for wages and employment. In short, we strive to provide an answer to the question whether firms are averse to uncertainty about a worker’s future performance and if so what the implications for employment and wages are.

Lazear (1998) shows that firms would prefer the risky workers if wages were not sensitive to performance and if workers could be fired. The intuition is straightforward: Take two workers with identical expected productivity. Make the distribution function of future productivity have a higher

1The literature on statistical discrimination describes how firms attempt to use a workers’ observable attributes to estimate his expected productivity (see Phelps, 1972). In this paper we assume that the expected productivity level is the same for all workers and analyse the effect of differences in the variance of future productivity.
variance for one than the other, that is one is a riskier hire than the other. Now we can monitor performance and fire the risky worker if he under-performs but retain him if he does well. Since his wages are independent of performance, the expected profits from hiring the risky worker exceed those from hiring the more predictable one. It follows that the risky hire becomes more profitable the sooner the worker’s performance is discovered, the younger he is and the easier it is to fire him. But note that the underlying assumption is that wages not follow performance, the risky hire is not going to be rewarded for good performance. We relax this assumption and allow wages, quite reasonably in our view, to follow productivity. We find that even with relaxing this assumption, the conclusion in Lazear (1998) continues to hold.

2. Profit maximisation

We assume that labour is the only factor of production and workers can only be hired at a cost, making them a quasi-fixed asset (see Oi, 1962). Moreover, we assume that the number of firms is finite due to the presence of fixed entry costs. The representative firm’s production function is assumed to be concave^2

\[ f(E) = A \log(E) \] (I)

where \( A \) is the aggregate productivity coefficient and \( E \) denotes the number of efficiency units of labour. We capture worker heterogeneity by assuming that workers may differ in the future in terms of the number of efficiency units of labour that they embody. In contrast to Lazear (1998)

^2 Alternatively, one could add physical capital to get diminishing returns to labour.
we assume that wages follow productivity. In particular, we assume that firms decide on the level of wages per efficiency unit of labour \( w \) and the hiring rate, \( h \). These are the representative firm’s two choice variables.

In the absence of uncertainty, the percentage change in employment, measured in efficiency units of labour, \( E \), is given by the hire rate \( h \), minus the quit rate, \( q \).

\[
\frac{\dot{E}}{E} = h - q \tag{2}
\]

We assume that although workers post-training productivity is known, the identity of the quitters is not known ex-ante, only the expected quit rate. It follows that the dynamics of employment are governed by the following stochastic equation where \( dE \) denotes the increment of a Wiener process and \( \sigma^2 \) denotes the variance of the density function for future performance or productivity:

\[
dE = [(h - q)E]dt + \sigma \sqrt{h + \phi q}E \, dW \tag{3}
\]

We let \( \phi \in (0,1] \) measure the importance of uncertainty about the quality of workers who quit and take this to measure the extent to which individual workers are complements in production, that is to what extent one worker’s productivity depends on the average level of productivity among his colleagues (see Lucas, 1988).

We then have, using Ito’s Lemma, the Bellman equation that describes the representative firm’s optimisation with respect to the number of hires and the real wage offered,

\[
\rho V = \max_{h,w} \left\{ \left( E - wE - T(h)E \right) + V_E \left( h - q \right) E + \frac{1}{2} \sigma^2 V_{EE} \left( h + \phi q \right) \right\} \tag{4}
\]

\( V \) denotes the value of the firm, \( \rho \) is the real rate of interest and the convex function \( T(h) \) denotes training costs per efficiency unit of labour, all of which are paid by the firm. The first three terms on the right-hand side represent the immediate profits from employing \( E \) efficiency units of labour whereas the remaining two terms represent the continuation value.

The first order condition for \( h \) is

\[
V_E E + \frac{1}{2} \sigma^2 V_{EE} E^2 = T'(h)E \tag{5}
\]

The two terms on the left-hand side show the marginal benefit of increased hiring while the term on the right-hand side shows the marginal cost. Uncertainty enters through the last term on the left and is due to uncertainty about the ability of a new worker. This is negative which shows that uncertainty reduces the rate of hiring. The result depends on the concavity of the production function.

Now suppose training costs can be described by a training cost function \( T(h) = Ae^{\alpha h} \). This implies that at \( h = 0 \) we have \( T(h) = A \), which implies that a fixed cost is incurred to maintain the capacity to train new workers.

We now have a solution for the optimal hiring rate,

\[
h = \frac{1}{\mu} \log \left( \frac{1}{\mu A} \right) + \frac{1}{\mu} \log \left( V_E + \frac{1}{2} \sigma^2 V_{EE} E \right) \tag{6}
\]

An increase in hiring costs \( \mu \) decreases hiring. The optimal rate of hiring is also a function of the marginal value of workers \( V_E \) taking into account uncertainty about their abilities as before.

We now assume the following functional form for the quit rate \( q = \alpha - \beta \log(w/w^*)R \) where \( \beta \) denotes a worker’s exogenously given quit propensity. The term \( R \) could either be exogenous or otherwise be a function of the rate of employment.\(^3,4\)

The quit rate is a negative function of the ratio of the firm’s own real wage to the average real wage elsewhere, \( w^* \), and \( \alpha \) is the quit rate when \( w = w^* \). Taking the derivative with respect to the real wage gives the following first order condition,

\[
-\left[ V_E + \frac{1}{2} \sigma^2 V_{EE} E \right] \frac{dq}{dw} = 1 \tag{7}
\]

or,

\[
w = \beta \left[ V_E - \frac{1}{2} \sigma^2 V_{EE} E \right] \tag{8}
\]

The quitting propensity \( \beta \) enters in a multiplicative manner as a key determinant of wages, which implies that if workers’ propensity to quit did not depend on the wage, the optimal wage would be zero.

Uncertainty enters the determination \(^8\)As in Calvo (1979).

\(^9\)We get a horizontal wage curve if \( \alpha \) is a constant and independent of the rate of employment.
of the wage in two ways. First, as shown below, the first derivative term is lower because of uncertainty about who will quit in the future. Second, the second derivative term makes the optimal wage higher because higher wages reduce quits and there is uncertainty about the ability of replacement workers. For the functional forms considered here, we will find the second effect to be stronger so wages are unambiguously higher because of uncertainty.

Substituting the two first order conditions and the production function into the Bellman equation gives the following equation

\[
\rho V(E) = \Lambda \log(E) - \beta \left[ V_{EE} - \frac{1}{2} \sigma^2 \phi V_{EE} E^2 \right] - \alpha' V_{EE} + \frac{\alpha'}{2} \sigma^2 \phi V_{EE} E^2 + \beta \left[ \log \left( \beta \left( V_{EE} - \frac{1}{2} \sigma^2 \phi V_{EE} E^2 \right) \right) \right] \times \left( V_{EE} - \frac{1}{2} \sigma^2 \phi V_{EE} E^2 \right) - \frac{1}{\mu} \left( V_E + \frac{1}{2} \sigma^2 V_{EE} E \right) E + \left( V_{EE} + \frac{1}{2} \sigma^2 V_{EE} E^2 \right) \times \left( \frac{1}{\mu} \log \left( \frac{1}{\mu} \Lambda \right) + \frac{1}{\mu} \log \left( V_E + \frac{1}{2} \sigma^2 V_{EE} E \right) \right)
\]

(8)

where \( \alpha' = \alpha + \beta \log(w/w^R) \). In the appendix we show that the Bellman equation has the solution,

\[
V(E) = \kappa + \lambda \log(E)
\]

(9)

where

\[
\lambda = \frac{\Lambda}{\rho + \beta \sigma \phi (1 + \frac{1}{\rho} (1 - \frac{2}{\rho}) \rho)}
\]

(10)

and \( \kappa \) is a complicated constant. The marginal value of a worker \( V_E \) is:

raised by a positive productivity shock, lowered by a rise in interest rates, lowered by an increase in quitting propensity \( \beta \), lowered by a rise in the contribution of each quit to the variance of the change in employment \( \phi \), and increased by a rise in hiring costs \( \mu \). Note that wages are independent of productivity when \( \mu = 0 \) as in Lazear (1998) and the dependence of wages on the other variables listed above vanishes.
3. Effect of Uncertainty on Hiring

Using the results of equation (6) and equation (10), we now investigate how hiring is affected by uncertainty,

\[
 h = \frac{1}{\mu} \log \left( \frac{1}{C^{22}} \right) \\
 + \frac{1}{C^{22}} \log \left[ \frac{(1 - \frac{C^{18}}{C^{19}}) C^{2}}{\rho + \beta + \frac{C}{C^{30}} (1 - \frac{C^{25}}{C^{27}})} \right]
\]

(11)

Hiring is a positive function of the aggregate productivity coefficient, \( \Lambda \), and a negative function of the real interest rate \( \rho \), hiring costs \( \mu \), the propensity to quit \( \beta \), and the importance of quit-related uncertainty, \( \phi \). The impact of increased uncertainty is more difficult to assess. Taking the derivative of (11) with respect to \( \sigma^2 \) gives

\[
\frac{\partial h}{\partial \sigma^2} = \frac{1}{1 - \frac{C^{16}}{C^{17}}} < 0
\]

(12)

The first term is the effect of uncertainty about hire quality. This leads to a reduction in hiring. The second term is the effect of uncertainty about the ability level of those who quit on the marginal value of a worker. The sum of these two terms is unambiguously negative. Thus, hiring is reduced by uncertainty.

4. Effect of Uncertainty on Wages

We can use equation (7) and equation (10) to obtain an explicit solution for the wage,

\[
w = \frac{\beta}{E} \left( 1 + \frac{\phi}{C^{2}} \sigma^2 \right) \frac{\Lambda}{\rho + \frac{C}{C^{30}} \sigma^2 + \frac{C}{C^{26}} + \frac{1}{\mu} (1 - \frac{C^{25}}{C^{27}})}
\]

(13)

The real wage is a positive function of aggregate productivity and training costs and a negative function of the real interest rate \( \rho \), hiring costs \( \mu \), the importance of quit-related uncertainty, \( \phi \), and the impact of increased uncertainty is more difficult to assess. Taking the derivative of (13) with respect to \( \sigma^2 \) gives

\[
\frac{\partial \log(w)}{\partial \sigma^2} = \frac{\beta}{\beta + \frac{C}{C^{2}} \sigma^2} - \frac{\beta \sigma^2}{\beta + \frac{C}{C^{2}} \sigma^2 + \frac{1}{\mu} (1 - \frac{C^{25}}{C^{27}})}
\]

(14)

The first term is the first line captures the importance of reducing quits to prevent existing workers, whose quality is certain, from quitting. The second term describes the negative effect of added uncertainty on the marginal value of existing workers. This would make the wage lower. The derivative is positive because of the inequality in...
5. Effect of Uncertainty on Steady-State Employment

We are primarily interested in how uncertainty affects the rates of employment and unemployment in steady state under conditions of equilibrium, which in the present context translates into equality of wages across firms. In steady state the following equality must hold,

\[ e_h = e_q \]  

(15)

which in equilibrium gives the following equation using the quit function and the solution for the optimal hiring rate in equation (11),

\[ R^{\text{opt}} E^{\uparrow} = \left( \frac{1}{\rho A} \right)^{\frac{1}{2}} \left( \frac{(1 - \phi^2) A^2 e^{-\sigma}}{\rho + \frac{1}{2} \phi^2 + \beta + \frac{1}{2} (1 - \phi^2)} \right) \]

(16)

Thus,

\[ \frac{\partial \log(Z)}{\partial \sigma^2} = \frac{-\frac{1}{2} \left( \frac{E - \rho}{\rho + \frac{1}{2} \phi^2 + \beta + \frac{1}{2} (1 - \phi^2)} \right)}{\frac{1}{2} (1 - \frac{E}{\rho}) - \frac{1}{2} \left( \frac{E - \rho}{\rho + \frac{1}{2} \phi^2 + \beta + \frac{1}{2} (1 - \phi^2)} \right)} < 0 \]

(19)

The derivative is unambiguously negative.\(^7\) This is caused solely by the effect of uncertainty on the hiring rate. The quit is not affected because a higher wage in one firm is matched by a higher wage in all other firms. We found that the hiring rate is reduced because of uncertainty about the productivity of new workers and also because of uncertainty about which workers will choose to quit. The latter effect reduces the value of existing workers to the firm.

Employment is a positive function of the aggregate productivity coefficient and a negative function of the real interest rate and the propensity to quit.

Now taking \( R \) to be constant term, we define

\[ z = R^{\text{opt}} E^{\uparrow} \]

(18)

and then take the derivative with respect to \( \sigma^2 \),

\[ \frac{\partial \log(Z)}{\partial \sigma^2} = \frac{-\frac{1}{2} \left( \frac{E - \rho}{\rho + \frac{1}{2} \phi^2 + \beta + \frac{1}{2} (1 - \phi^2)} \right)}{\frac{1}{2} (1 - \frac{E}{\rho}) - \frac{1}{2} \left( \frac{E - \rho}{\rho + \frac{1}{2} \phi^2 + \beta + \frac{1}{2} (1 - \phi^2)} \right)} < 0 \]

(19)

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\[ \text{6} \text{ An additional mechanism to create higher wages, which is not incorporated in our model, is lower turnover through an income effect; if firms pay workers who are a good match higher wages, they are less likely to seek other work due to income effects and therefore firms can afford to pay higher wages.} \]

\[ \text{7} \text{ The same applies if we let } \sigma \text{ depend on } E, \text{ now denoting average form employment, such as: } R = E^2. \]
6. Conclusions

This paper has extended the model of Lazear (1998) by allowing for endogenous wages set by firms. Our results are supportive of his results and provide interesting microeconomic foundations. We have shown that firms prefer paying workers higher wages when their future productivity is more uncertain—be it because of personal issues or the nature of business and technology developments—because firms prefer lower hiring rates and lower employment levels in this case, which raise marginal productivity. In addition, firms are willing to pay higher wages in order to reduce quitting and avoid risky replacements. What makes hire rates lower is, first, a lower value of trained employees because of uncertainty about which workers may quit in the future, and, second, uncertainty about the quality of new hires.

Appendix

In this appendix, we show that the solution of the turnover-training model is,

$$V(E) = \kappa + \lambda \log(E)$$  \hspace{1cm} (20)

where $\kappa$ and $\lambda$ are constants to be determined. We first show that equation (9) satisfies the Bellman equation and prove that it is unique and well behaved. Plugging the posited solution, equation (9), into the Bellman Equation and noting that,

$$V'_{E} = \lambda$$

$$V_{E} + \frac{1}{2} \sigma^2 V_{EE} = \frac{\lambda}{E} \left[ 1 - \frac{\sigma^2}{2} \right]$$  \hspace{1cm} (21)

it follows that the pair $(\kappa; \lambda)$ satisfies:

$$\rho V(E) + \rho \lambda \log(E) = \Lambda \log(E) - \beta \lambda \left( 1 + \frac{\sigma^2}{2} \right) \phi' \lambda' - \frac{\phi' \sigma}{2} \phi \lambda$$

$$+ \beta \log \left( \beta \left( 1 + \frac{\sigma^2}{2} \right) / E \right) \left( \lambda' \left[ 1 + \frac{\sigma^2}{2} \right] \phi \lambda \right)$$

$$- \frac{\lambda}{E} \left( 1 - \frac{\sigma^2}{2} \right) + \lambda \left( 1 - \frac{\sigma^2}{2} \right)$$

$$\times \left( \frac{1}{\mu} \log \left( \frac{1}{\mu} E \right) + \frac{1}{\mu} \log \left( \lambda \left[ 1 - \frac{\sigma^2}{2} \right] / E \right) \right)$$  \hspace{1cm} (22)

Equating terms of equal order results in two equations in two unknowns $(\kappa, \lambda)$. Since equation (22) is non-linear in $\lambda$ this looks very hard. However, the equation is linear in $\log(E)$ so that this can be used to determine $\lambda$. Once we have solved for $\lambda$, we can substitute the solution into the equation determining $\kappa$.

Thus,

$$\rho \lambda = \Lambda - \beta \lambda \left( 1 + \frac{\sigma^2}{2} \right) - \frac{\lambda}{\mu} \left[ 1 - \frac{\sigma^2}{2} \right]$$  \hspace{1cm} (23)

which upon simplifying results in the solution for $\lambda$ given by equation (10). Since $\kappa$ enters the equation for the constant terms linearly and is the only unknown after $\lambda$ is determined, the solution for $\kappa$ is unique.

We now show that the posited solution is unique and satisfies the transversality condition. For simplicity, we will focus on cases in which hire and quit rates are bounded. We will then derive conditions under which these
assumptions are valid. By boundedness, there exists a constant \( k \) such that,

\[ |h - q| \leq K \]

(24)

uniformly. Then, rewriting the equation for employment (3) as,

\[ dE = \alpha(E, t)dt + \beta(E, t)dW \]

(25)

It follows that there exist other constants \( K_1 \) and \( K_2 \) such that,

\[ |\alpha(E', t) - \alpha(E, t)| + |\beta(E', t) - \beta(E, t)| \leq K_1 |E' - E| \]

(26)

and

\[ |\alpha(E, t)|^2 + |\beta(E, t)|^2 \leq K_2 (1 + |E|^2) \]

(27)

It then follows that at any \( t \), employment is a square-integrable process with continuous sample paths (Karatzas and Shreve, 1988, Thm. 2.9, p. 289). Furthermore, by the same theorem, there exists a constant \( C \) such that,

\[ E(E_t^2) \leq C(1 + E_t^2)e^{Ct} \]

(28)

uniformly.

From the solution for the value function, equation (9), and using Jensen's Inequality we also get,

\[ E[V(E_t)] \leq \kappa + \lambda \ln E(E_t) \]

(29)

Using equation (28), it follows that there exist constants \( C_1 \) and \( C_2 \) such that,

\[ E[V(E_t)] \leq C_1 + C_2 t \]

(30)

which implies the transversality condition,

\[ \lim_{t \to \infty} e^{-at}V(E_t) = 0 \]

(31)

Using the Bellman verification theorem (Fleming and Soner, 1993, p. 172-173), it follows that the candidate solution is unique.

It remains to derive conditions under which our boundedness assumptions are valid. We shall assume that some or all of the parameters in the hiring and quit functions depend on aggregate labour market conditions. These are captured by the variable \( R \), which is a function of the average level of employment. Firm cannot individually influence this market variable.

For the hire rate, equation (11), we define

\[ \Psi = \log(\mu) - \log \left( \frac{1 - \sigma^2}{2} \right) - \log(A) + \log \left( \rho + \beta + \frac{\beta \sigma^2}{\mu} \right) \]

(32)

If,

\[ 0 \leq \mu h \leq \mu K \]

(33)

it follows that

\[ 0 \leq \log(E) - \log(A) - \Psi + K\mu + \log(E) \]

(34)

Thus

\[ e^\Psi \leq \frac{E^{-1}}{E} \leq e^{\psi + K\mu} \]

(35)

so that training costs, \( A \), have to vary inversely with employment per firm, \( E \). Note that since \( K \) can be very large, this relationship might be extremely weak.

For the quit rate, we have similarly that,

\[ 0 \leq \alpha + \beta \log(R) \leq K \]

(36)

so that assuming \( \beta \) is a constant, \( \alpha \) needs to vary roughly inversely with \( \log(R) \) at large values of \( R \). These conditions are considerably less restrictive if we assume \( E \) to be bounded away from zero and infinity.


