Actuarial Discretion vs. Rules in
Regulation of Pension Funds

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J. MICHAEL ORSZAG

1. INTRODUCTION

Solvency regulations for pension funds companies share many characteristics with Value at Risk (VaR) analyses. For instance, the U.K. Minimum Funding Requirement (MFR), introduced as part of the Pensions Act of 1995, requires pension funds whose funding falls below 90% of their immediate liabilities to raise their capital levels and to undergo more frequent MFR solvency evaluations. This paper examines rules vs. discretion (e.g., internal models) in valuation regulation of pension funds. This is particularly relevant because model development in this field is largely dominated by actuaries who have their own internal models; yet, regulators often use fixed rules which may not include risk-based capital.

While there are similarities between regulatory and performance measurement issues for investment banks and long-term business, there are also many differences. First, because pension funds deal with long-term business, the short-run VaR framework is not entirely appropriate as myopic asset-liability matching does not take into account the long-run considerations which are crucial for pension analysis. Instead, a more appropriate framework may be that of ruin theory (c.f., (Daykin, Pentaikainen and Pesonen 1994)) which takes into account the probability of becoming insolvent any time in the future.

Second, the special feature of pension funds is that their liabilities are basically not marketable. While a pension fund which winds up can fulfill its obligations through contracting for a deferred annuity with
an insurance company, many of these liability transfers involve significant long-term risks which depend on aggregate factors and hence are difficult to market. There are of course examples: Legal & General purchased the pension obligations of 7,000 members in the Maxwell Communications Work scheme for £80m (Pensions Week, 24 October 1997). Anecdotal evidence also indicates that the failure of some U.K. insurance companies to accurately predict improvements in mortality has led to serious problems among suppliers of deferred annuities.

In addition, the tax treatment of pensions is a special regulatory feature in that governments wish to restrict pension fund surpluses to limit tax reliefs paid. Hence, pension regulators need to control both solvency and surplus. At the same time, some countries such as Germany allow benefits to be built up as accounting book reserves. While this regulation is typically not risk-based, it could be (as tax authorities normally want to minimize the probability of providing excess tax relief).

The issues in this paper are relevant to other long-term business such as insurance. For example, the U.S. National Association of Insurance Commissioners’ early warning system allows or dictates various levels of regulatory intervention as risk-based solvency levels fall (Klein 1995). The issue of nonmarketability of liabilities is also relevant: long-term risks from changes in health or longevity involve far more balance sheet risk than short-term catastrophe risks such as from flooding or earthquakes, the bulk of reinsurance transactions is for short-term property and casualty liabilities, not long-term life liabilities. In addition, as with pensions, actuaries dominate life insurance valuation and their assumptions and models are prominent in regulation of this business. Nevertheless, to keep the focus manageable, we have chosen to focus on pensions specifically.

2. ANALYTICAL FRAMEWORK

To address these issues as clearly as possible, it is useful to write down as simple as possible a valuation model.\textsuperscript{1} The following equation

\textsuperscript{1}This formula is presented to illustrate conceptually the main points. In practice, actuarial formulae are somewhat more complex and may account for spouse’s benefits and a variety of discretionary benefits. A more complete formula is presented in Appendix B.
captures the liabilities of a firm to a worker in a final salary scheme who entered a scheme at time \( t_{k-1} \) at age \( t \):²

\[
(1) \quad P_{PUM}(t_{k-1}, t) = a(t - t_{k-1})W(t)R(t, t_N)A(t_N)D(t, t_N)
\]

where:
- \( a \) – the accrual rate (typically \( \frac{1}{60} \)),
- \( t_{k-1} \) – the age at entry into the scheme,
- \( t \) – the current age of the scheme member,
- \( t_N \) – the normal retirement age of the scheme member,
- \( W(t) \) – the pensionable salary at age \( t \),
- \( R(t, t_N) = [(1 + g)(1 + \pi)]^{t_N-t} \) – the revaluation factor describing how benefits are uprated between ages \( t \) and \( t_N \),
- \( g \) – the growth rate of real wages.
- \( A(t_N) \) – the annuity factor (the present value of a pension annuity of £1 per annum) at retirement age \( t_N \) (typically lies between 12 – 16),
- \( D(t, t_N) \) – the discount factor (\( \left( \frac{1}{1+r} \right)^{t_N-t} \) if the discount rate \( r \) is constant).

The present value (at the time of leaving) of the corresponding pension benefits payable from retirement age is found by revaluing the accrued benefit to retirement age by the revaluation factor \( R(t, t_N) \), capitalizing the subsequent stream of pension payments (multiplying by \( A(t_N) \)) and then discounting this capitalized sum back to the leaving date (multiplying by \( D(t, t_N) \)).

The valuation method in Eq. (1) is the Projected Unit Method. It projects wages to retirement but in calculating liabilities does not assume the worker will work any more years at the firm. Hence, the pension fund does not accrue assets to cover the extra years of work of the worker at the firm and the contribution rate is rising steeply with age, increasing the cost of hiring older workers.

This is of course not the only possible way to fund a pension scheme, but it is used by about 75% of UK pension schemes and, except for the retirement wage assumptions, is the same as the method prescribed for MFR funding calculations and for transfer value (cash equivalent) calculations as implemented in the Institute and Faculty of Actuaries Guidance Note 11 (GN11) ((Institute and Faculty of Actuaries 1998)).

²We focus on final salary schemes as these are still the preeminent forms of occupational pension provision. A similar analysis could be done for career revalued occupational schemes; however, with a few minor exceptions (e.g., the UK plumbers industrywide scheme), these have not really caught on.
An alternative would be to use ‘prospective benefit funding method’ such as the ‘attained age method’ (AAM). This method sets the contribution rate taking into account future salaries and service and hence, if expectations about these are realized, arrives at a constant, age-independent marginal contribution rate. We can determine the constant contribution rate \( c \) needed to generate the same pension in retirement as given by the Projected Unit Method by setting:

\[
N = \frac{P_{PUM}(t_0, t_N)}{\sum_{s=t_0}^{t_N} W(s)(1 + r)^{t_N-s}}. \tag{2}
\]

With the MFR norms satisfied, the standard contribution rate is 12%. Fig. (1) compares the constant contribution rate with the upward sloping contribution rate implicit in the PUM. With the PUM, the effective contribution rate is initially much lower than the AAM contribution rate, equals it in mid-career and is more than double the constant contribution rate at retirement. The slope of the upward sloping contribution rate flattens out slightly at retirement as the discount rate is lowered on a 10 year sliding scale from equity yields to gilt yields. This factor is already taken into account in determining the constant contribution rate.

Eq. (2) shows that the constant contribution rate using the AAM is given by the ratio of the full service pension at retirement age (using the PUM) to the compound value of lifetime earnings. So long as the interest rate \( r \) in Eq. (2) is the same as the realized yield on pension fund assets (as will be the case if the MFR norms are satisfied), there will exist an ‘equivalent’ defined contribution pension scheme that will also generate the same pension in retirement if the annual contribution rate into this scheme is also set at \( c \).

\[\text{This formula follows from equating the value of the attained age pension fund with that of the PUM fund at the retirement date } t_N.\]

\[\text{This contribution rate depends on the choices of annuity factor and accrual rates. However, schemes which provide spouse’s benefit and discretionary benefits will have higher contribution rates to compensate for these additional benefits. This calculation uses the assumptions in Appendix A and assumes an annuity factor of 12.5.}\]

\[\text{By definition, the defined contribution scheme cannot build up surpluses or deficits. However, if the realized yield on the portfolio of assets differs from } r, \text{ then a surplus or deficit will build up in a defined benefit scheme, and under the AAM (or other similar prospective benefit methods) the contribution rate will be adjusted to eliminate the surplus or deficit.}\]
We will simply calculate the value of pension fund liabilities by adding up the liabilities over all members and assume that all members of a given age are identical. The implication of the rising marginal contribution rate is that, although actuaries using the PUM to value aggregate pension liabilities of a scheme set a single contribution rate (as a proportion of earnings) for all members whatever the age, the

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One reason this is inappropriate is that cross-subsidies between different classes of members are a crucial part of the analysis of pension schemes. Regulation in most countries (Netherlands is an exception) is such that in most cases, early leavers subsidize long stayers and penalties for changing jobs can be severe; we have calculated a loss of the order of up to 30% of pension benefits for a typical worker over the course of his career (Blake and Orszag 1997). Another significant cross subsidy is that workers with steep wage profiles have pensions subsidized by those with flat wage profiles.
effective cost to the scheme of employing older members is higher than that of younger members.

3. Regulation

European countries differ quite a bit in how they implement rules for calculating individual pension liabilities in Eq. (1) as well as the demographic assumptions which lead to aggregate liabilities. Countries which are absolutely prescriptive are the Netherlands and Germany. Other countries allow assumptions to vary in some range: Luxembourg is an example as is Belgium. Typically most restrictions are put on the discount rate. Many countries prescribe the use of standard mortality and disability tables. These different rules are summarized in detail in (Collinson 1993).

The U.K. Inland Revenue has a maximum funding level for tax purposes as well as Minimum Funding Requirement (MFR) regulations which were introduced with the Pensions Act of 1995. The MFR regulations require salary related schemes to be able meet their liabilities on a specific valuation basis. Pension schemes must undergo a MFR valuation every three years; funds that do not meet 90% of MFR must meet that level within a year and schemes which are severely underfunded may be required to undergo emergency valuations. Schemes with assets between 90% and 100% of MFR levels have five years to make up a shortfall. The Pensions Act of 1995 came into law on 6 April 1997 but schemes are given ten years from then (until 5 April 2007) to fully meet MFR norms and until 5 April 2003 to achieve 90% solvency (c.f.,((Hammond Suddards Solicitors 1995), Ch. 7), for more details). The MFR rules cover most active private occupational defined benefit schemes in the U.K.7

The MFR rules are reasonably prescriptive. They specify for instance that everyone retires at a normal retirement age; specify the assumed returns on bonds, index-linked gilts, equities; the difference in age between husband and wife; transfer value calculations; and even expenses to be used on winding up a scheme. The specific MFR assumptions are described in Appendix A. Assets are valued at market basis but liabilities are adjusted to reflect dividends relative to long-run equilibrium levels. Inland Revenue are similarly prescriptive: they

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7 Certain defined benefit schemes are excluded from the MFR such as death benefit only schemes, public sector schemes covered by government guarantee and schemes with fewer than two members.
specify for instance an interest rate of 8.5%, salary increases of 6.9%, and pension increases of up to 6.4%, and mortality using PA(90) down-rated one year.

There are a number of problems with this sort of regulation. First, the rules are not risk-based. The actual portfolio does not matter at all for solvency calculations. Second, they do not account for variations in circumstances of firms. Third, they rely on rigid assumptions based on outdated data; the PA(90) tables are extrapolations based on data collected in the late 1970s (MacDonald 1996).

3.1. **Market Value Adjustments.** Significant evidence of this later problem is the use of Market Value Adjustments (MVA). The MVA to be used for members more than ten years from MFR pension age is the equity MVA. For those less than ten years before retirement, the MVA used should be calculated by a linear combination of equity and gilt MVAs assuming a progressive switch to a 100% gilt investment from a 100% equity investment (GN27, 3.11).

**Equity MVAs**

For equities, the MVA is the ratio of 4.25% to the gross dividend yield on the FT-SE Actuaries All-Share Index on the date that the MVA is calculated: Fig. (2) plots this MVA from 1976 to 1996. Fig. (3) plots the equity MVA for 1990 to 1996. The arithmetic mean from 1976 to 1996 was 0.9147, the minimum was 0.513 and the maximum was 1.49. Thus, over this period, the average effect of the MVA was to reduce pension liabilities. On the other hand, from 1990 to 1996, the average was 1.002, so that on average the MVA was neutral; the minimum over this period was 0.725 and the maximum was 1.32. From 1994 to 1996, the average was 1.1099.

The reduction in dividend tax credits in the U.K. coupled with the risk in the stock market effectively have lowered the net dividend yield, making the market adjustment on dividends much larger than 1. Because the MVA multiplies liabilities, this increases liabilities relative to assets, increasing the required return on stocks and creating MFR deficits for schemes. Since the Inland Revenue rules do not use Market Value Adjustments, it is plausible but not probable that pension funds could at the same time be in deficit by MFR rules but required by Inland Revenue to reduce their surpluses.
4. The Role of Actuarial Discretion

One obvious way to address the contradictions and problems with prescriptive regulation of pension funds is to allow pension funds to use their own internal models. This would allow better risk management and attention to firm-specific circumstances.

The problem with this is most clearly illustrated by a Coopers and Lybrand survey in 1996 where the average actuary allocated £100 to fund a given liability (Coopers & Lybrand Pensions News, May 1996). Here, different actuarial assumptions led to results of between £29 and £148 for funding a given liability. Even government rules lead to assumptions which differ widely: the new terms for contracting out of the state pension system (SERPS) led to a need for £110 in funding whereas the minimum funding requirement regulations led to a need for £93. Assumptions about funding levels are far from innocuous not only because they have direct affects on perceived solvency and security but also because such assumptions are used directly in determining cash equivalents and transfer values for workers who change jobs.

Figure 2. Market value adjustment on equities, 1976-1996.
Figure 3. Market value adjustment on equities, 1990-1996.

Understanding the role of actuarial discretion is essential for reliance on internal models in regulation. The principal elements of calculations subject to discretion are:

- The choice of discount factor in computing present values.
- The choice of inflation rate in revaluing projected wages.
- The choice of annuity factor. GN27 (D.1) suggests the PA90 mortality tables downrated two years, at least for smaller schemes (less than £100 million of pension liabilities). However, GN11 does not place any constraints on the actuary outside of Minimum Cash Equivalent calculations and, even in this case, GN11 allows the actuary to vary the morality assumptions, e.g. they are permitted to use unisex mortality factors.\(^8\) In cases where a pension would be payable only to the current spouse, the spouse’s age may be used (GN11, 4.5) and this may affect the annuity factor.
- Projected increase in wages.

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\(^8\) According to the Government Actuary’s Department, the majority of UK schemes use unisex mortality factors.
• Marital status. MFR norms specify proportions married but the actuary in Minimum Cash Equivalent calculations can make use of the actual marital status of the spouse.

• Discretionary benefits. Both the old and new versions of GN11 allow for variations due to additional discretionary benefits in cash equivalent calculations. Discretionary benefits may be franked against the Minimum Cash Equivalent (which does not take into account discretionary benefits).

With the exception of the last two items, all are included in Eq. (1). To assess the effectiveness of actuarial discretion, we compute elasticities of Eq. (1) with respect to the various parameters the actuary can choose. For example, suppose we wish to consider how liability estimates change when the discount rate assumption changes by a small amount, then we compute the elasticity of the projected unit liability with respect to the discount rate:

\[
\eta_r = \frac{\partial P_{PUM}(t_0, t)}{\partial r} \frac{r}{P_{PUM}},
\]

and evaluate the other parameters in this expression at the MFR norms.\(^9\)

The effect of actuarial discretion depends naturally on the age of the worker as well as the years of service. The elasticities are summarized in Table (1).

These tables show that the discount rate, annuity factor, inflation rate and wage growth rate assumptions are crucial in actuarial calculations. Thus, different actuaries making different assumptions about these values can produce dramatically different calculations. For example, if the assumed value of the discount rate deviates by 11% from

\(^9\)The other elasticities are:

\[
\eta_r = (t_N - t) \frac{\pi}{1 + \pi}
\]

\[
\eta_r = -(t_N - t) \frac{r}{1 + r}
\]

\[
\eta_g = (t_N - t) \frac{g}{1 + g}
\]

\[
\eta_A = 1.0
\]
Orszag, *Actuarial Discretion vs. Rules*

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<th>Actuarial Parameter</th>
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**Table 1.** Elasticities of pension liabilities with respect to actuarial discretion.

the MFR norms so that, for example, a discount rate of 10% is used instead of the MFR basis of 9% for a worker aged 35 with 30 years to retirement, Table (1) indicates that the pension cash equivalent is reduced by about 27.5%.

It is useful to convert these elasticities of individual liabilities into aggregate liabilities.\(^{10}\) In Table (2), we report elasticities for aggregate pension liabilities assuming a uniform age distribution from ages 25 to 65.

\(^{10}\)

We define the *liability weighted duration*:

\[
< t_N - t > = \frac{\sum (t_N - t)f(t)P_{PUM}(t)}{\sum f(t)P_{PUM}(t)}
\]

where \(f(t)\) is the number of (identical) members of age \(t\). In this case, the elasticities are:

\[
\eta_\pi = < t_N - t > \frac{\pi}{1 + \pi}
\]

\[
\eta_r = - < t_N - t > \frac{r}{1 + r}
\]

\[
\eta_g = < t_N - t > \frac{g}{1 + g}
\]

\[
\eta_A = 1.0
\]

Using MFR assumptions and a uniform population distribution between ages 25 and 65, we have a liability weighted duration of 8.1415.
Yet, even official government assumptions differ considerably. We have compiled some recent assumptions in Table (3).\textsuperscript{11} Here, we note that the maximum inflation figure is 25\% larger than the smallest, whereas the range in discount rates is 5.9\%, the range in male longevity is 7.3\%, the range in female longevity is 10.6\% and the range in real wage growth is 33\%. Table (2) suggests that differences in inflation assumptions alone would lead to about a 10\% spread in valuations for a scheme with a uniform age structure. Table (2) also suggests the importance of mortality assumptions. The annuity factor suggested by Inland Revenue numbers for men is 7.91 whereas that dictated by the MFR is 8.37; the corresponding figures for women are 8.98 and 9.50. If a scheme members are evenly split between men and women, this suggests that mortality assumption differences between using MFR norms and Inland Revenue norms in a projected unit valuation would lead to roughly a 6\% spread in liability valuations.

\textsuperscript{11}MFR figures are from ((Institute and Faculty of Actuaries 1998), B27.11-12), Inland Revenue figures are from ((Ure 1998), p. 45), GAD figures are from (Government Actuary’s Department 1996).
Having demonstrated the significance of actuarial discretion, we proceed to analyze specific factors involved in actuarial discretion in more detail.

4.1. **Discount Factor.** The old version of GN11 stated in sec. 3.2:

... actuarial value should be assessed having regard to market rates of interest. One of the ways in which a market value assessment may be made is on the basis of market redemption yields on British Government Stocks of appropriate duration and type at the time of transfer with allowance for investment of future interest receipts at such rates as the actuary considers reasonable. In valuing benefits which are subject to revaluation in accordance with the general index of retail prices, yields on index-linked gilts will be an appropriate criterion.

When the yield is higher, the discount factor used in computing cash equivalents is higher and the transfer value is correspondingly lower. By using longer-term bonds and thereby higher yielding bonds, transfer values are thus reduced more for younger members. Transfer values would be reduced yet further if yields of securities with even higher returns such as equities were used. GN11 now permits the yields on these higher yielding securities to be used in computing transfer values:

It is a fundamental requirement, stemming from legislation, that a cash equivalent should represent the actuarial value of the benefits which would have otherwise have been preserved. Such actuarial value should be assessed having regard to the market rates of return on equities, gilts or other such assets as the actuary considers appropriate...

The direct consequence of the new version of GN11 and the use of higher discount factors is the the lowering of pension liabilities as well as the cash equivalents received by workers who change jobs. The discount rate enters the calculation of the transfer value in two ways: it affects the annuity factor since a rise in the discount rate reduces the discounted value of future pension payments, and it changes the rate at which the deferred pension is discounted to the date on which the transfer value is paid.

The 9% figure is a reasonable one to use in computing equity yields. It is difficult to estimate accurately the mean growth rate of equities; however, the average annual return on equities between 1955 and 1995
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Table 4. Dependence of liability calculations on actuarial assumptions.

was about 14\% with an annual standard deviation of about 30\% (Barclays de Zoete Wedd 1996). The figure of 14\% however should be adjusted downwards to capture a risk adjustment for holding equities and 9\% is not an unreasonable risk-adjusted figure. Similarly, the average gilt yield between 1955 and 1995 of 8.05\% is very close to the MFR assumption. The discount factor 30 years from retirement can differ by a factor of 3 under plausible assumptions concerning the discount rate. However, it is unclear that the equity discount rate (instead of that on indexed bonds) is the most appropriate one to use since deferred pensions also contain an element of insurance over inflation.

4.2. Inflation Revaluation Factor. The law requires that the deferred pension is uprated using LPI, i.e., the minimum of the rate of inflation and 5\% p.a. compound. Hence, if the average rate of inflation exceeds 5\% p.a. compound between the leaving and retirement dates, the deferred pension is not fully indexed. Before the mid-1970s, there were no indexing requirements and early leavers lost much of the value of their pension due to high inflation. The historical average annual appreciation of the RPI in the U.K. between 1955 and 1995 was 6.74\%.

12\footnote{The formula for the discount factor in this case: \((1 + \text{Discount rate } #1)^{-20} (1 + \text{Discount rate } #2)^{-10}.\}
but inflation appears to have slowed down in the past decade. The MFR norm stated in GN27 is 4% per annum. Table (5) shows the dependence of the calculated pension liability on the actuarial assumption used.\(^\text{13}\)

### Annuity Factor

For older workers, liability calculations are most sensitive to the actuarial calculation of the annuity factor. The annuity factor is the present value of an annuity of £1 per annum beginning at the retirement age, taking into account survivorship probabilities and any uprating of the annuity over time to account for inflation. The relevant formula is:

\[
A(t_N) = \sum_{s=1}^{\infty} \left[ \frac{1 + \hat{\pi}}{1 + \hat{\mu}} \right]^s sP_{t_N}
\]

(5)

\[
= \sum_{s=1}^{\infty} \left[ \frac{1}{1 + \hat{\mu}} \right]^s sP_{t_N}
\]

where:
- \(sP_{t_N}\) – survivorship probability from age \(t_N\) to \(t_N + s\).
- \(\hat{\pi}\) – the annual uprating factor.
- \(\hat{\mu}\) – the nominal discount rate.

\(^{13}\)This table is computed using the formula: \(\left[ \frac{1 + \pi}{1 + 0.04} \right]^R\) where \(\pi\) is the assumed inflation rate and \(R\) is the number of years to retirement.
The MFR rules require that survivorship probabilities are based on the mortality tables PA90 downrated two years (to account for the improvements in mortality since these tables were constructed). The PA(90) tables are suitable for the type of people taking out private pension annuities at retirement age in the sense that such people are likely to enjoy greater longevity than the population as a whole (for which another set of mortality tables such as the English Life Tables No. 14 might be more appropriate).

Table (6) lists the annuity factors for men and women at different retirement ages; they also show the life expectancies of these individuals (these were found using Eq. (5) with $\hat{\rho} = 0$). The calculations

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</tbody>
</table>

\[ \tilde{\pi} = 0\% \quad \tilde{\pi} = 3.5\% \quad \tilde{\pi} = 4\% \]

Table 6. Life expectancy and annuity factors at retirement age based on PA90.

$\hat{\rho}$ – the real discount rate (defined as $\frac{1+r}{1+\tilde{\pi}} - 1$)
were made using the Standard Tables Programme (Continuous Mortality Investigation Bureau 1994). The nominal discount rate \( r \) used was 8% which is the same as the MFR basis for retired workers (namely, the yield on gilts). Three different assumptions concerning uprating were made, namely 0%, 3.5% and 4.0%, to assess the sensitivity of the annuity factor. For a 65 year old man, the effect of a 0.5% absolute change in the inflation assumption is a 5% change in the value of the annuity factor, demonstrating the sensitivity of pension liabilities to assumptions underlying the annuity factor.

5. Real Wage Growth Rates

Wage growth is a special assumption because under current U.K. legislation future real wage growth by workers who change jobs. When a worker opts for a frozen, deferred pension, s/he loses subsequent real wage growth. In taking a transfer value to a new scheme, a worker loses future wage growth in excess of the assumed rate. Thus, the assumptions on real wage growth used in inward transfers are crucial in assessing portability loss.

Adopting the MFR assumption of a constant real wage growth ignores the following variations:

- **Real wage growth differs between men and women.** Real wages in manufacturing have grown at approximately 2% as shown in Fig. (4). The median real wage growth for men for 1984-1996 was 1.7% per year and the median real wage growth rate for women for 1984-1996 was 2.6%. The mean real wage growth for men for 1984-1996 was 2.07%, whereas for women it was 2.90%.\(^{14}\)

- **Real wage growth varies across industries and occupations.** The median manual male worker only had average real wage growth of 1.1% between 1984 and 1996, whereas the average non-manual male worker had double this growth rate. Even among manual workers, there is an industry-specific dispersion in growth rates. For example, the average male manual worker in paper and tobacco, printing and publishing (1980 SIC 47) exhibited average nominal wage growth of 5.5% p.a. whereas in construction (1980 SIC 50), it was approximately 6.5% p.a.

\(^{14}\)The source for these calculations is the New Earnings Survey (1984 data from Department of Employment 1985), 1994 data from (Office of National Statistics 1994). The price index used was the average for the year.
Typical wage profiles do not have constant growth rates but rather are likely to have higher real wage growth at the beginning of a career and lower or even negative real wage growth near retirement. Furthermore, wage profiles differ between men and women. According to the New Earnings Survey, the average gross weekly pay of those aged 50-59 in 1996 was about 6% lower than that of those aged 40-49, whereas the gross weekly pay of those aged 30 to 39 was 22% higher than that of those aged 25 to 29. These figures, taken at a single point of time, do not take into account the fact that the worker who is in the age range 25 to 29 will very likely be earning even more than his colleagues who are currently aged 30 to 39 when he reaches that age range because of real wage growth.
6. TOWARDS INTERNAL MODELS FOR MARKET VALUE ADJUSTMENTS

In the previous section, we have reviewed the sensitivity of actuarial calculations to long-run assumptions. At the same time, the rigidity of regulations which fix long-run assumptions impose constraints on the ability of pension funds to manage risk. For example, the current MVA rules discourage foreign investment and raise liabilities in stock market booms so as to create pressure for very high returns and risky investments at the very time when the stock market is at its highest point.

A simple solution here is that the market value adjustment could be calculated on a risk-adjusted basis; funds which choose to have less risky investments would face lower thresholds for increasing their contributions and hence would in effect have lower capital requirements. The problem with this is that it ignores the correlation structure between pension assets and liabilities: wages, job tenure and other liability-specific factors have significant cyclical components.

Another and we believe a better potential approach is to allow internal models but check the models by comparing five year contribution schedules with those needed to meet historical incremental liabilities calculated using retrospective market values. Schemes whose accrued contribution schedules produced assets not sufficient to meet these incremental liabilities would face more stringent tests in the future form of a higher multiplier on liabilities. Similarly, schemes whose accrued contribution schedules had assets above these incremental liabilities using historical data would face lower multipliers on liabilities.

For example, suppose that the model assumed wage growth of 3% and the actual wage growth was 4% then the contribution schedule all else equal would have produced too small a sum of accrued assets to meet liabilities and therefore, not only would the scheme be required to raise contributions to meet the shortfall but it also would face a higher liability multiplier in the future.

Such an approach is far from free of problems. However, as a matter of practice, no amount of regulation can eliminate discretion entirely and it is useful to have in place some incentive mechanisms for using the correct model. Furthermore, as the population ages and nears retirement, risk and therefore risk-based capital becomes more important both for firms and individuals.
This appendix contains the MFR norms exactly as they appear in the appendix: ‘Current Factors for Use in MFR Valuation’ in Guidance Note 27 of the Faculty and Institute of Actuaries (Institute and Faculty of Actuaries 1998), B27.11-12

A. The current gilt yields to be used for valuing pensioner liabilities should be the gross redemption yield on the FT-Actuaries Fixed Interest 15 year Medium Coupon Index or the FT-Actuaries Index-linked Over 5 years (5 % inflation) Index, as appropriate. In the case of LPI pension increases, either fixed-interest gilts with 5% pension increases or indexed-linked gilts assuming pension increases 0.5% less than inflation should be used, whichever gives the lower value of liabilities. Similar principles should be applied for other pensions which are index-linked but subject to a cap other than 5%.

B.1. The long-term financial assumptions to be used are as follows:
- Rate of inflation – 4 % per annum
- Effective rate of return on gilts – 8 % per annum
- Effective rate of return on equities - pre MFR pension age – 9 % per annum
- Effective rate of return on equities - post MFR pension age – 10 % per annum
- Rate of increase of GMP under Limited Revaluation – 5 % per annum
- Rate of statutory revaluation for deferred benefits – 4 % per annum
- Rate of LPI increase in payment – 3.5 % per annum
- Rate of increase in post 1988 GMPs – 2.75 % per annum
- Rate of increase in S148 Orders – 2 % per annum

The real rate of return on index-linked stocks is \( i \) where \( (1 + i) = \frac{1.08}{1.05} \).

B.2. An additional assumption needed for the projection calculations for the Schedule of Contributions.
- Rate of salary growth – 6 % per annum.

C. Market Value Adjustments (MVAs)

C.1. The MVA in relation to equities should be the ratio of 4.25% to the gross dividend yield on the FT-SE Actuaries All-Share Index.

C.2. The MVA in respect to gilts should be the value at the annualised yield on the FT-Actuaries Fixed Interest 15 year Medium
Coupon Index or the FT-Actuaries Index-linked Over 5 years (5% inflation) Index, as appropriate, of a 15 year stock with coupon equal to the relevant long-term assumption, payable annually in arrears.

C.3. For liabilities which when in payment might be valued using either the yield on a fixed-interest gilt basis or that on an index-linked gilt basis, the MVA should be that which produces the lower liability.

C.4 If the liability includes a retirement lump sum payment, for the lump sum liability the market value adjustment on the proportion \(g\) of that part of the liability deemed invested in gilts (e.g. 0.3. if seven years from MFR pension age) should be:

\[
\{1 - (1 - g) \cdot (1 - \text{gilt MVA})\}
\]

D. Demographic assumptions

D.1. Mortality (before and after retirement) – PA90 rated down two years.

D.2. In the case of schemes which have a pensioner liability (assessed on the gilt basis) of at least £100 million, the mortality basis to be adopted should be that which the actuary considers appropriate for that scheme in respect of current pensioners and other members who have reached MFR pension age. In the case of all other schemes, and for non-pensioners below MFR pension age, the standard mortality table specified above should be adopted.

D. 3. Proportions married. For pensioners, the assumption should be consistent with 80% (men) or 70% (women) at age 60. For non-pensioners, the assumption should be, at the assumed date of retirement or earlier death, 80% (men) or 70% (women).

D. 4. Age difference between husband and wife – 3 years.

E. Expenses.

E. 1. The allowance to be made for expenses connected with closure of the scheme, continuation as a closed scheme and eventual wind-up should be 4% of the value of the accrued liabilities for the first £50 million of such liabilities, 3% of the value of the accrued liabilities for the next £50 million of such liabilities and 2% of the remainder of the value of accrued liabilities. For this purpose money purchase liabilities are only included in the value of accrued liabilities, if the scheme rules state that expenses cannot be met out of assets attributable to those liabilities.
In this appendix, we present the general formula for calculating pension liabilities by the projected unit method to account for such factors as survivorship probabilities to retirement age, discretionary benefits, death-in-service and spouses’ benefits. With the additional factors, Eq. (1) for the value of pension benefits becomes:

\begin{equation}
P_{MU}(t_{k-1}, t) = i_{N-t}P_{MU}(t_{k-1}, t) \left[ 1 + S(t, t_N) \frac{A_S(t_N)}{A(t_N)} \right] + \hat{D}S(t, t_N)
\end{equation}

where:

\begin{align*}
P_{MU}(t_{k-1}, t) &= a(t - t_{k-1}) W(t) R(t, t_N) A(t_N) D(t, t_N) \\
&= a(t - t_{k-1}) W(t) [(1 + g)(1 + \pi)]^{t_N-t} A(t_N) D(t, t_N)
\end{align*}

where:

- \(a\) – the accrual rate (typically 7.5 but the U.K. MFR reference scheme has 8.0),
- \(t_{k-1}\) – the age at entry into the scheme,
- \(t\) – the current age of the scheme member,
- \(t_N\) – the normal retirement age of the scheme member,
- \(W(t)\) – the pensionable salary at age \(t\),
- \(R(t, t_N) = [(1 + g)(1 + \pi)]^{t_N-t}\) – the revaluation factor describing how benefits are uprated between ages \(t\) and \(t_N\),
- \(g\) – the growth rate of real wages (MFR norm 2%),
- \(\pi\) – the inflation rate (MFR norm 4%),
- \(r\) – the discount rate (MFR norm 8% – 10%),
- \(D(t, t_N)\) – the discount factor \(\left[\frac{1}{1+r}\right]^{t_N-t}\) if the discount rate \(r\) is constant,
- \(A(t_N)\) – the annuity factor (the present value of a pension annuity of £1 per annum at retirement age \(t_N\) (typically lies between 12 – 16),
- \(S(t, t_N)\) – the spouse’s pension as a fraction of the total pension multiplied by the probability married. The MFR norms assume that 80% of men and 70% of women are married when they reach the MFR pension age. The MFR reference scheme in the Pensions Act of 1995 does not include a spouse’s pension but does include a widow’s/widower’s pension of half the member’s pension. As an example, if the 80% rate is applied to all spouses, \(S(t, t_N) = 0.80 \times 0.50 = 0.40\). For cash
equivalent (transfer value) calculations, GN11 allows the actuary to use the actual marital status of the member (GN11, 4.5).

\( A_S(t_N) \) – the annuity factor for the spouse. The MFR norms state that \( A(t_N) \) and \( A_S(t_N) \) are to be calculated according to PA(90) down-rated two years. However, some variation is permitted such as assuming that \( A(t_N) = A_S(t_N) \). The MFR norm specifies an age difference between husband and wife of three years. The spouse’s actual age may be used for Minimum Cash Equivalent purposes only where the pension would be payable to the current spouse (GN11, 4.5). The MFR norms permit large schemes (with liabilities in excess of £100 million) to use other mortality bases (e.g., English Life Tables, etc.).

\( D_S(t, t_N) \) – value of discretionary benefits, death-in-service, and disability benefits. For death-in-service benefits, the MFR reference scheme in the Pensions Act of 1995 (sec. 136(12B)) provides the spouse with a pension of 50% of the pension due to the member if s/he had been a deferred pensioner at the retirement age of the spouse. The Faculty and Institute of Actuaries’ GN11 directs that discretionary benefits be taken into account in computing cash equivalents unless the trustees direct otherwise.

In Minimum Funding Requirement calculations, liabilities are then multiplied by a market value adjustment term as specified in GN27 (c.f., Appendix A).

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